Pricing Investor Impact*

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Abstract

This paper presents a novel asset pricing model that allows for investor impact, defined as the change in non-market (e.g. social and environmental) outcomes generated by an investor's actions. Price elasticities of supply and demand determine contribution multipliers for each asset, representing the degree to which shifts in investor demand lead to shifts in supply, and hence, investor impact. Heterogeneity in the price elasticities implies considerable cross-sectional variation in the contribution multipliers and covariance with expected financial returns. If the potential for investor impact is correctly reflected in asset prices, it should not be possible for investors with impact preferences to shift their demand in a way that improves their utility. Using this principle, a theoretical valuation formula for investor impact is derived along with the associated optimal portfolios. An empirical calibration finds a 3% contribution multiplier for the average large and medium-sized US stock, suggesting potential for significant investor impact even in these liquid assets, but making it challenging to reconcile observed ESG-driven demand shifts with impact preferences. Nevertheless, divestment may be coherent with impact preferences in select cases. Ideal impact investments are large, positive allocations to profitable, socially productive assets with elastic supply and inelastic demand. These findings have implications for how the pursuit of investor impact is managed, researched and regulated.

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In sustainable investment, "investor impact" is defined as the change in non-market (e.g. social and environmental) outcomes generated by an investor's actions (Brest et al., 2018; Kölbel et al., 2020; Busch et al., 2021). Although assessing changes relative to counterfactual scenarios presents a formidable challenge, high-profile impact-oriented investors, such as development finance institutions, are routinely expected to demonstrate their "additionality"—that is, to provide evidence that their investments generate investor impact (Carter et al., 2021). Underscoring its significance, a recent consensus of over 3,000 practitioner organizations identified "investor contribution" (a synonym for investor impact) as a key dimension for impact management (Boiardi and Stout, 2021; Impact Frontiers, 2023).

In her 2023 presidential address to the American Finance Association, Starks (2023) called for more theoretical frameworks that consider the differences between investors with different nonpecuniary preferences or "values". I respond to this call with an asset pricing model that allows investors to have "impact preferences" with an explicit concern for aggregate non-market outcomes, as opposed to "holdings-based preferences" that depend only on the characteristics of the assets held by an investor[1]. In contrast to the exogenous tilts emphasize in the literature on holdings-based preferences, I show that investors with impact preferences develop endogenous portfolio tilts that can be dramatically different. These tilts are shaped not only by exogenous nonpecuniary characteristics, but as much or more by firm and market characteristics which determine the potential for investor contribution.

This paper addresses three central questions: First, how can investor impact be formally defined within an asset pricing model? Second, what is the optimal policy for an investor with investor impact as one of their goals? Third, what effects on asset prices, non-market outcomes, and investor utility should be expected from investors pursuing these optimal policies?

To answer these questions, this paper develops an asset pricing model based on a supply and demand system with a single, aggregate "non-market good"[2]. The model, and the

[1]See Green and Roth (2021) and Edmans and Kacperczyk (2022) for further discussion. Heinkel et al. (2001) and Pástor et al. (2021) are prominent examples of models with holdings-based preferences. In general, such preferences include any utility that depends on asset holdings, including due to subjective financial beliefs. When related to sustainable investing they are also known as "values-aligned" or "warm glow" preferences in reference to the work of Andreoni (1990).

[2]This paper is neutral on what this good might be. For example, the good could be excludable or rival (i.e. not a "public good") or only relevant to market participants (i.e. not an "externality"). As such I use this general term that covers any good that is not directly traded on the market.

extensions I explore, draws on insights from papers across multiple stream of literature, principally including Koijen and Yogo (2019), Van der Beck (2022), Betermier et al. (2022), Pástor et al. (2021), and Merton (1987). The model features many heterogeneous firms and agents, yet it is highly tractable and yields intuitive expressions. All investors internalize the investor impact of their portfolio choice because assets are subject to non-trivial supply and demand curves.

The primary channel for investor impact that I consider is capital allocation affecting the scale of firms' production. Firms vary in the degree of their "socially productivity", that is the amount their production contributes to the non-market good. The degree to which a shift in demand leads to a shift in supply (and hence production of the non-market good) is naturally captured by "contribution multipliers" which are depend on the supply and demand elasticities for each asset. To first order, the contribution multiplier $\mathcal{C}$ that defines the ratio between an investor i's demand shift ΔK_i and the resulting net shift in supply of the target firm ΔK is given by

$$\mathcal{C} \equiv \frac{\Delta K}{\Delta K_i} \approx \frac{\zeta_S}{\zeta_D + \zeta_S},$$

where ζ_D is the price elasticity of demand and ζ_S is the price elasticity of supply. These endogenous contribution multipliers are similar in function to the exogenous decision weights in Hart and Zingales (2017).

The model nests many traditional asset pricing models which typically make assumptions that make these contribution multipliers less relevant or uninteresting. For example, the assumption of inelastic supply ($\zeta_S = 0$), which has long been standard in the literature on asset pricing in endowment economies (Lucas, 1978), implies that the contribution multipliers will all be zero. In contrast, I show that if supply is even slightly elastic then the contribution multipliers can be economically significant and exhibit substantial cross-sectional variation.

Having defined investor impact and demonstrated its dependence on contribution multipliers, I next derive the optimal strategy for an investor with impact preferences. Contrary to conventional wisdom in the literature, in my model even infinitesimal, uncoordinated investors internalize the contribution of their portfolio choice to the non-market good. In essence, infinitesimal demand shifts generate infinitesimal price shifts which lead to infinitesimal impacts if supply is elastic. Such shifts can be cost-effective from the perspective of an independent investor who values impact. This contrasts with Pástor et al. (2021) and related papers where inelastic supply means that investors do not internalize the effect of their choices on the non-market good, or Oehmke and Opp (2020) where investor coordination is required[3].

I show that the main difference between the optimal "impact tilt" and the tilt associated with holdings-based preferences is that the impact tilt depends on a matrix of contribution multipliers. The impact tilt can be understood in terms of "impact returns" that stand alongside expected financial returns in determining optimal portfolios. These impact returns, while endogenous in that they depend on the supply and demand for each asset, can be inserted as exogenous tilts into the models of papers like Pástor et al. (2021), Pedersen et al. (2021), and Zerbib (2022) so the key insights of these papers for portfolio choice still hold.

I next extend the model to allow for passive investors as in Betermier et al. (2022). The effect of passive investment is critical as it means that active investors, whose changes in demand drive changes in supply, can have leveraged effects as passive investors follow along. Without accounting for this effect, my empirical estimates of the contribution multipliers would be substantially lower.

Following these relatively general developments, I next study the supply and demand elasticities that arise in a microfounded specification of the model. I use a standard mean-variance specification for investor utility and define firm utility as depending on the firm's market value minus the costs of investment. A novel feature that I introduce here is that firms' costs of investment depend on the amount of investment by other firms via a cost-sensitivity matrix. This is necessary to generate non-zero supply cross-elasticities. Intuitively it is a way to account for competition and diminishing returns to scale at the industry level. This microfounded case allows me to show that cross-sectional variation in demand and supply elasticities should be expected to generate a relatively positive covariance between expected financial and impact returns, in line with the findings of Cole et al. (2020) for the IFC's portfolio.

Next I discuss how the "price of impact", the strength of investors' taste for investor impact, can be empirically determined. First, it can be inferred from observed impact tilts after accounting for the non-market outcomes generated by firms and the associated contribution multipliers. Second, while ultimately the price of impact is a subjective parameter,

[3]See also the related discussion in the appendix of Green and Roth (2021).

if investors follow a coherent strategy, then social costs and charitable opportunities can be used to inform upper bounds on its value. That is, they should not be willing to pay more for impact with their investments than the cost-effectiveness of the best marginal charitable opportunity in their wider opportunity set. Otherwise they will be paying more for impact than necessary, ultimately decreasing their utility.

After presenting my model, I use it as a framework to study the contribution multipliers and prices of impact that can be inferred from the relevant empirical literature. First, I leverage the results of Betermier et al. (2022) to estimate an average contribution multiplier of 3% associated with large- and medium-sized stocks in the US. This contribution multiplier is high enough to be economically relevant (Betermier et al., 2022), yet small enough to be in line with the weak to null results so far from studies that examine the link between investor demand shifts and real impact on corporate operations (Berg et al., 2023; Noh et al., 2023; Briere and Ramelli, 2022). Note that the effects observed by Briere and Ramelli (2022) are greater for firms with lower credit ratings (more inelastic demand) in line with my model.

Second, to illustrate the potentially dramatic variation in contribution multipliers across assets (and asset classes), I present a table of expected contribution multipliers for firms with a range of different supply and demand characteristics. This builds on a table from Merton (1987). These results highlight the potential heterogeneity in the cross-section and that supply elasticities are just as important as demand elasticities. The right supply elasticity can produce low contribution multipliers even for cases with relatively inelastic demand or high contribution multipliers even for cases with relatively elastic demand.

Third, based on results from Koijen et al. (2022), Noh et al. (2023), and Betermier et al. (2022), I infer the prices of impact implied by observed impact returns for various ESG metrics after accounting for the estimated average contribution multiplier. I first consider the price that investors appear willing to pay per dollar of revenue to improve a firm's ESG performance by one standard deviation. For the log-emissions intensity metric of Noh et al. (2023), I also infer a price of impact per ton of greenhouse gas (GHG) emissions. The prices of impact are all of the same order of magnitude and all surprisingly, if not implausibly, high. This is a first point of tension between observed tilts and impact preferences.

Finally, I infer "bottom-up" prices of impact from reported social costs and charitable cost-effectiveness estimates for GHG emissions. The prices of impact I report range over two orders of magnitude. Yet, these prices of impact are all lower than the prices of impact inferred from observed ESG tilts, adding to the tension between these tilts and impact preferences.

There are several possibilities that could resolve this tension, and I highlight these potential explanations as directions for further research. First, the investors in the empirical sample may not be pricing impact in a way that aligns with my model. This could be because impact tilts are more driven by emotional considerations than careful analysis, as observed by Heeb et al. (2022). Or they may be rationally extracting impact-related benefits that are not about the impact of their asset allocation (e.g. via shareholder engagement). Second, some investors may see ESG scores as financially informative and tilt towards higher scoring firms purely to maximize their financial utility. Third, the empirical results I have leveraged in this paper may not be valid for my application in some non-trivial way. For example, the estimated ESG score coefficients might be driven by firms whose characteristics imply a high contribution multipliers. This would conflict with my use of the cross-sectional average contribution multiplier in my calculations. If so, the solution would require further refinement of the already impressive empirical methodologies employed in the relevant papers—this is out of scope for this paper.

I next present two short extensions to my baseline model that demonstrate how contribution multipliers arise in additional settings. First, the possibility of investor influence on endogenous firm social productivities is an important part of many sustainable investors "theory of change" and has been, for example, studied in Pástor et al. (2021). The attraction of this impact channel is that even if supply is inelastic, investor commitments to pay an impact premium may induce firm management to invest in improving their social productivity. I show that this may be an important channel for impact, depending on firm characteristics, but that in any case the potential for impact via this channel will be moderated by an elasticity-dependent contribution multiplier. Second, I consider a simple model with search frictions where only one investor invests per firm and I show that a supply and demand-dependent contribution multiplier also arises in this case.

The overall contribution of this paper is to show how models that account for heterogeneity in the supply and demand for capital can be used as a rich framework to answer important questions that go beyond traditional asset pricing. This includes questions related to blended finance and universal ownership. I conclude the paper with a discussion of such potential applications and extensions.

This paper is related to several literatures. The focus on impact preferences in my model complements work on asset pricing models with exclusion or holdings-based preferences (Pástor et al., 2021; Berk and Van Binsbergen, 2021; Zerbib, 2022; Pedersen et al., 2021; Baker et al., 2022a; Fama and French, 2007; Avramov et al., 2021a; Avramov et al., 2021b; Goldstein et al., 2022; Luo and Balvers, 2017; Friedman and Heinle, 2016; Gollier and Pouget, 2022; Barnea et al., 2005; Heinkel et al., 2001). My results explicitly linking investor portfolio tilts to contribution multipliers complement other models that have a non-market good but where contribution multipliers are fixed by assumption or are not explicitly disentangled (Green and Roth, 2021; Oehmke and Opp, 2020; Baker et al., 2022b; Piatti et al., 2022; De Angelis et al., 2022; Broccardo et al., 2022; Gupta et al., 2021; Hong et al., 2021; Landier and Lovo, 2020; Chowdhry et al., 2019; Roth, 2021). The combined tilting and divestment strategy of Edmans et al. (2022) aligns naturally with my model—a difference is that in my model any investor may adopt this strategy, not just a blockholder.

My use of a supply and demand system highlights both the importance of the demand curve literature (Noh et al., 2023; Koijen et al., 2022; Koijen and Yogo, 2019; Van der Beck, 2021; Van der Beck, 2022; Merton, 1987) and the importance of models that allow for an endogenous supply side (Betermier et al., 2022; Bai and Zhang, 2022; Choi et al., 2021; Gonçalves et al., 2020; Bai et al., 2019; Ma, 2019; Belo et al., 2013). In general my model can be applied as a framework to any assets. For example, the large effects found to be associated with the provision of credit in Green and Vallee (2022), Banerjee and Duflo (2014), and Blouin and Macchiavello (2019) are consistent with the asset markets in question featuring high supply elasticity and low demand elasticity.

My results also relate to the growing empirical literature that demonstrates sustainability-related preferences are driving investor decisions, fund flows and willingness-to-pay (Riedl and Smeets, 2017; Hartzmark and Sussman, 2019; Barber et al., 2021; Bauer et al., 2021; Heeb et al., 2022; Siemroth and Hornuf, 2021). Green and Roth (2021) review the evidence for whether asset owners have impact preferences or not. They find mixed evidence, ranging from fund marketing that suggests a desire to appeal to impact preferences, to Heeb et al. (2022)'s findings that are in line with holdings-based preferences, to evidence of impact preferences being important in some studies of charitable giving. I relate the price of impact in my model to charitable opportunities and social costs, complementing other models with charitable opportunities including Graff Zivin and Small (2005), Baron (2007), and Baron

(2009). See Emerson (2003), among other papers, for an early example of the application of social returns to impact investing. This paper also contributes a generalized asset pricing model to the as yet small stream of literature that analyzes optimal strategies for altruistic investors from first principles (Roth Tran, 2019; Harris, 2021a; Harris, 2021b; Baker et al., 2022b).

My findings are in line with the arguments of Berk and Van Binsbergen (2021), Broccardo et al. (2022), and Davies and Van Wesep (2018) that divestment is unlikely to be a high-impact strategy, yet add the nuance that select cases of divestment may be coherent depending on an investor's price of impact. The shape of an ideal investment that emerges from my model—investments in highly profitable, highly socially productive firms with inelastic demand and elastic supply—matches the features of the investments made by investors with impact preferences as found by Cole et al. (2022) and Kovner and Lerner (2015). My model for impact returns offers an approach to answering their call for analysis of if the non-market outcomes generated by impact investing are sufficient to compensate for the lower financial returns found in the literature. Contribution multipliers are also a more nuanced approach to assessing additionality than the binary approach they used (i.e. did an investment involve commercial investors or not).

This paper is organized as follows. Section 1 presents my baseline model for a generalized supply and demand system. Section 2 presents the microfounded case. Section 3 discusses the price of impact. Section 4 presents estimates of contribution multipliers and prices of impact based on a review of relevant empirical results. Section 5 develops extensions of the model that illustrate how contribution multipliers also arise in other cases. Section 6 discusses the limitations of the model and potential further extensions and Section 7 concludes.

1 The Model

In this section, I introduce the model that serves as a framework for the rest of the paper. Vectors and matrices are in bold and their elements are indicated in parentheses (e.g. $x(n, m)$ is the nth row and mth column of matrix $\boldsymbol{x}$). If $\boldsymbol{x}$ is a vector, then $diag(\boldsymbol{x})$ is the diagonal matrix with $\boldsymbol{x}$ on the diagonal, otherwise it is the vector of diagonal values of the matrix $\boldsymbol{x}$. I denote the identity matrix as $\boldsymbol{I}$.

1.1 A supply and demand system

My model is a generalized supply and demand system that resembles the demand system of Van der Beck (2022) with the addition of an elastic supply side.

There are I investors indexed by $i \in \{1, \ldots, I\}$, N firms indexed by $n \in \{1, ..., N\}$, and a single period between two times $t \in \{0, 1\}$.

Initially, at time 0, each firm has installed capital $K_0(n) \geq 0$ (after depreciation) and no other resources or liabilities. There is one share outstanding for each unit of installed capital and each investor initially holds $K_{i,0}(n)$ shares with $K_0(n) = \sum_{i=1}^{I} K_{i,0}(n)$. The firm funds investment and generates an economic profit for its initial shareholders by simultaneously increasing its installed capital and shares outstanding to $K(n) = K_0(n) + \Delta K(n)$, selling all shares at price $P(n)$, and distributing the resulting cash flow $P(n)K(n) - I(n)$ proportionally to the initial shareholders. Here $I(n)$ is the cost of investment to install $\Delta K(n)$ of new capital. $K(n)$ may be interpreted as the firm's book value (in which case $P(n)$ is the market-to-book ratio of the firm).

Each firm also produces a contribution $G(n) = g(n)K(n)$ to the non-market good. This may be viewed as a first-order approximation to the true dependence of the non-market good on $K(n)$, with $g(n)$ an exogenous coefficient that defines the "social productivity" of the firm. The non-market good at time 1 is then $G = \sum_{n=1}^{N} G(n)$. I define G so that it is positive in the direction that is considered "good" (so, for example, positive CO2 emissions produce negative $g(n)$ and G).

Each investor's wealth at time 0 consists of their initial holdings in each firm, $\boldsymbol{K_{i,0}}$, and cash $C_{i,0}$. In equilibrium each investor's wealth is equal to $W_{i,0}$ and the total initial wealth is $W_0 = \sum_{i=1}^{I} W_{i,0}$. For the present period, investors may invest in the shares of the N firms, as well as a riskless asset with exogenous interest rate r_f.

Investors have heterogeneous beliefs and, as in Koijen et al. (2022), I assume that investors have full information about other investors' beliefs and agree to disagree. In particular, all agents are only required to agree about the market observables: prices $\boldsymbol{P}$, firm capital amounts $\boldsymbol{K}$, and the holdings of each investor $\boldsymbol{K_i}$. Investors may disagree about all other model parameters and I indicate this with a subscript i on such parameters as appropriate.

Each investor's demand curves $\boldsymbol{K}_i(\boldsymbol{P}, \boldsymbol{X})$ are a investor-specific function of the vector of current asset prices $\boldsymbol{P}$ and a collection of other model variables $\boldsymbol{X}$ (the choice of variables will depend on the microfoundations of specific implementations of the model). Aggregate demand is $\boldsymbol{K}_D = \sum_{i=1}^{I} \boldsymbol{K}_i$. Similarly, I stack each firm's supply curve into the function $\boldsymbol{K}_S(\boldsymbol{P}, \boldsymbol{X})$. Market clearing implies that total asset demand equals total supply:

$$\sum_{i=1}^{I} \boldsymbol{K}_i(\boldsymbol{P}, \boldsymbol{X}) = \boldsymbol{K}_S(\boldsymbol{P}, \boldsymbol{X}) \tag{1}$$

Each investor's demand curves have an associated *absolute* price elasticity of demand matrix, $\tilde{\boldsymbol{\zeta}}_i$, with elements $\tilde{\zeta}_i(n,m) \equiv -\frac{\partial K_i(n)}{\partial P(m)}$. I refer to the diagonal elements as elasticities and the off-diagonal terms as cross-elasticities. The aggregate absolute price elasticity of demand matrix is $\tilde{\boldsymbol{\zeta}}_D = \sum_{i=1}^{I} \tilde{\boldsymbol{\zeta}}_i$. Similarly, but with a different sign, the absolute price elasticity of supply matrix $\tilde{\boldsymbol{\zeta}}_S$ has elements $\tilde{\zeta}_S(n,m) \equiv \frac{\partial K_S(n)}{\partial P(m)}$. As discussed in Van der Beck (2022) the demand elasticity matrix can be thought of as a function of trading costs, cash flow correlations and risk aversion, or investment constraints. The supply elasticity matrix can be thought of as arising from capital adjustment costs and competition between firms to productively install capital and gain market share (assuming diminishing returns to scale at the sector or industry level). Section 2 presents a microfounded model for both sets of elasticities. Further understanding these drivers is an important avenue for future research.

As in Van der Beck (2022) it will be useful to be able to refer to "price multiplier" matrices, $\tilde{\boldsymbol{\mathcal{M}}}_S = \tilde{\boldsymbol{\zeta}}_S^{-1}$ and $\tilde{\boldsymbol{\mathcal{M}}}_D = \tilde{\boldsymbol{\zeta}}_D^{-1}$, that are the inverse of the elasticity matrices. The diagonals of these matrices are the supply and demand slopes.

The supply and demand curves are endogenous as, similar to Haddad et al. (2021), investors are sophisticated and strategically update their demand curves to be optimal for their own expected utility given their beliefs about aggregate demand and supply. The next proposition formally defines investor impact in terms of such strategic updates and shows how it can be approximated to first order.

Proposition 1.1. ***Investor Impact***. *The investor impact of investor i, defined as $II_i \equiv G(\boldsymbol{K}|\boldsymbol{K_i} + \Delta \boldsymbol{K}_i) - G(\boldsymbol{K}|\boldsymbol{K_i})$, is the change in the equilibrium expected value of the non-market good G when the investor strategically shifts their demand curve from $\boldsymbol{K}_i$ to $\boldsymbol{K}_i + \Delta \boldsymbol{K}_i$. To first order investor impact is given by*

$$II_i = \boldsymbol{g}' \boldsymbol{\mathcal{C}} \Delta \boldsymbol{K_i}, \tag{2}$$

where $\boldsymbol{\mathcal{C}} \equiv \tilde{\boldsymbol{\zeta}}_S(\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1}$ is the "contribution multiplier matrix". Equivalently, $\boldsymbol{\mathcal{C}} = (\tilde{\boldsymbol{\mathcal{M}}}_D +$

$\tilde{\boldsymbol{\mathcal{M}}}_S)^{-1}\tilde{\boldsymbol{\mathcal{M}}}_D$ based on basic matrix identities.

See Appendix A.1 for the derivation. The key that makes this definition tractable when each investor is small is that such demand shifts, by the fixed amount $\Delta\boldsymbol{K}_i$, don't affect the aggregate demand elasticity and hence don't cause the demand curves of other investors to update.

Figure 1 offers an intuitive visualization of investor impact for the case of a single firm. The contribution multiplier matrix, $\boldsymbol{\mathcal{C}}$, determines how much an investor's demand shift $\Delta\boldsymbol{K}_i$ contributes to $\Delta\boldsymbol{K}$ (and ultimately to the non-market good). Note that $\boldsymbol{\mathcal{C}}$ depends only on ratios between elasticities, not their levels. It is zero when either supply is assumed to be fully inelastic ($\tilde{\boldsymbol{\zeta}}_S = 0$) or demand is assumed to be fully elastic ($\text{diag}(\tilde{\boldsymbol{\zeta}}_D) \to \infty$). These are explicit or implicit assumptions in many models. However, when neither of these assumptions are imposed investor demand shifts can generate investor impact.

Investor impact could be generated by any shift in demand whatever the underlying intentions. Demand shifts can be strategically chosen with the goal of generating a positive investor impact. But they can also arise from changes to financial beliefs, transaction costs or other frictions and constraints. Thus, it could be that an investor's work to lower the transaction costs associated with investing in highly socially productive firm (for themselves and possibly others) does more to shift demand and generate investor impact than a consciously "altruistic" portfolio tilt.

It is important that I have defined investor impact based on the change in the investor's entire demand function, $\Delta\boldsymbol{K}_i$, not just in regards to the change with respect to a single firm. As long as the cross-elasticities are non-zero a shift in investor demand for one firm will have spillover effects on other firms. For example, as in Van der Beck (2022), spillover effects could include unintentionally increasing demand for firms with negative social productivity. Using this model I can calculate investor impact in a way that accounts for these non-trivial effects. These results offer guidance on how investor impact might be assessed in practice. The following proposition shows how investor impact, and the contribution multiplier matrix $\boldsymbol{\mathcal{C}}$, can be broken down into intuitive components.

Proposition 1.2. *The investor impact of a demand shift ΔK_i according to the beliefs of*

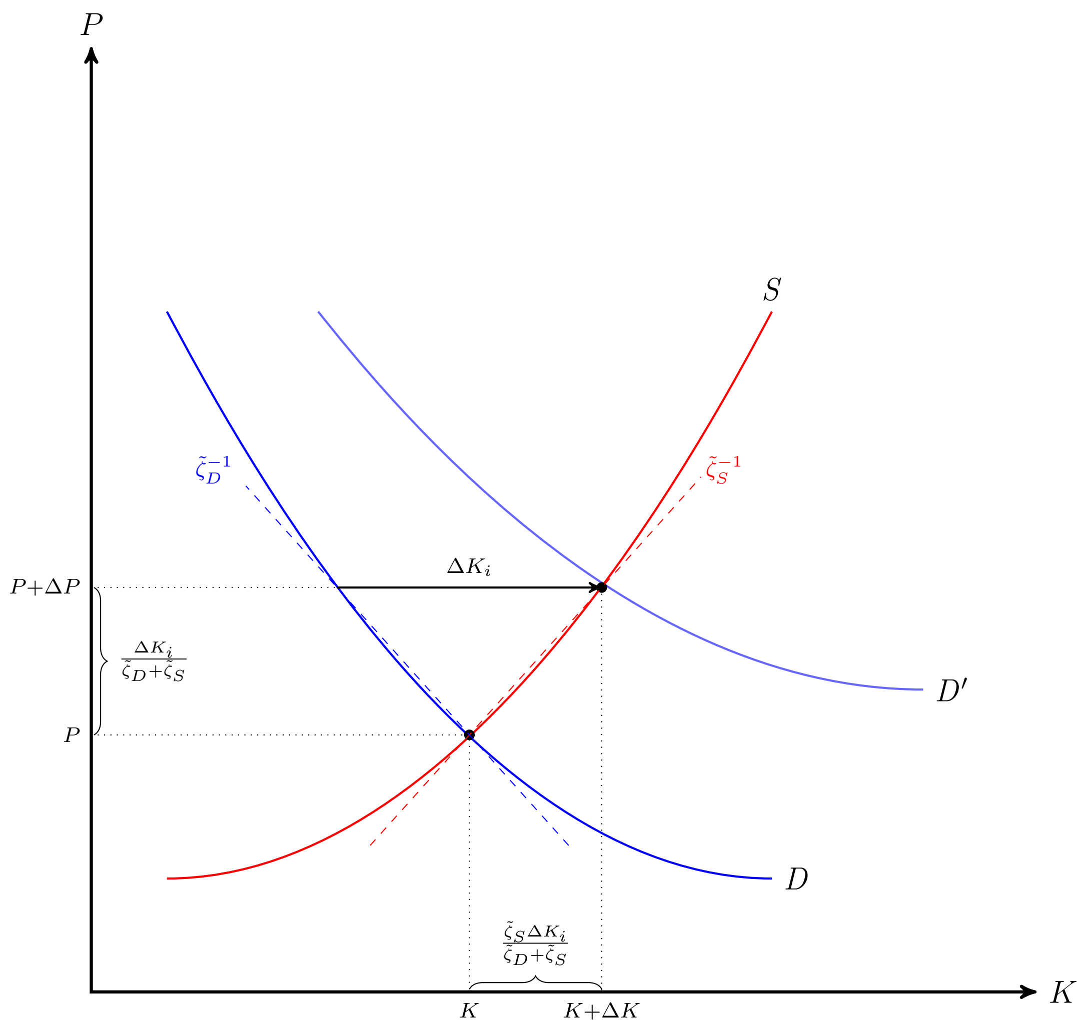

Figure 1: **Impact of a shift in an investor's demand curve**. This stylized diagram shows the effect of an individual investor shifting their demand curve for a single firm. All other investor's demand curves are assumed fixed so that the aggregate demand curve moves from D to D' based on the investor's chosen shift ΔK_i (which may be a function of P). Because of the point at which the market clears moves along the firm's fixed supply curve S. Near the equilibrium points, the demand curve's slope is approximately the inverse of the aggregate demand elasticity, $\tilde{\zeta}_D^{-1}$, and the supply curve's slope is approximately the inverse of the aggregate supply elasticity, $\tilde{\zeta}_S^{-1}$. The equilibrium moves from (K, P) to $(K+\Delta K, P+\Delta P)$ with price shift $\Delta P = \frac{\Delta K_i}{\tilde{\zeta}_D+\tilde{\zeta}_S}$ and net supply and demand shift $\Delta K = \mathcal{C}\Delta K_i$, where $\mathcal{C} = \frac{\tilde{\zeta}_S}{\tilde{\zeta}_D+\tilde{\zeta}_S}$ is the "contribution multiplier" .

investor i may be written as

$$II_i = \sum_{n=1}^{N} g_i(n)\mathcal{C}_i^E(n)\mathcal{C}^I(n)\Delta K_i(n), \tag{3}$$

with ***investor contribution multiplier*** $\mathcal{C}^I(n) \equiv \mathcal{C}(n,n) = \mathcal{C}_0^I(n)\mathcal{C}_{Cross}^I(n)$ *where*

$$\mathcal{C}_0^I(n) = \frac{\frac{\tilde{\mathcal{M}}_D(n,n)}{\tilde{\mathcal{M}}_S(n,n)}}{1 + \frac{\tilde{\mathcal{M}}_D(n,n)}{\tilde{\mathcal{M}}_S(n,n)}}, \tag{4}$$

and

$$\mathcal{C}_{Cross}^I(n) = \frac{\left(1 - \frac{\tilde{\boldsymbol{\mathcal{M}}}_D(n,-n)}{\tilde{\mathcal{M}}_D(n,n)}\boldsymbol{\beta}_{T,n}\right)}{\left(1 - \frac{\tilde{\boldsymbol{\mathcal{M}}}_T(n,-n)}{\tilde{\mathcal{M}}_T(n,n)}\boldsymbol{\beta}_{T,n}\right)}, \tag{5}$$

and with ***enterprise contribution multiplier***

$$\mathcal{C}^E(n) = \left(1 - \frac{\boldsymbol{g}'_{(-n)}}{g(n)}\boldsymbol{\beta}_{T,n}\right), \tag{6}$$

where $\tilde{\boldsymbol{\mathcal{M}}}_T = \tilde{\boldsymbol{\mathcal{M}}}_D + \tilde{\boldsymbol{\mathcal{M}}}_S$ *and* $\boldsymbol{\beta}_{T,n} = (\tilde{\boldsymbol{\mathcal{M}}}_T(-n,-n))^{-1}\tilde{\boldsymbol{\mathcal{M}}}_T(-n,n)$.

See Appendix A for the derivations. These results highlight both the basic intuitions behind investor impact and the potential complexities. Note that while I have presented the results in terms of the price multiplier matrices, because they offer a particularly intuitive interpretation, similar formulas can be derived in terms of the elasticity matrices.

$\mathcal{C}^I(n)$ is broken out as the investor contribution multiplier because it can intuitively be understood as determining the effect of the investor's demand shift $\Delta K_i(n)$ on installed capital $K(n)$. It also doesn't depend on $\boldsymbol{g}$, whereas the enterprise contribution multiplier $\mathcal{C}^E(n)$ does. Furthermore, in Appendix A.3, I show that the enterprise contribution multiplier may be interpreted as the ratio between a firm's "enterprise impact", defined as the change in the non-market good between an economy with and without the firm, and its "absolute impact" of $g(n)K(n)$.

The first part of the investor contribution multiplier, $\mathcal{C}_0^I(n)$, has a simple interpretation:

investor contribution is greater for firms with more elastic supply and less elastic demand.

To develop intuition for the other terms, it is useful to imagine the matrix $\tilde{\boldsymbol{\mathcal{M}}}_T$ as the covariance matrix of a set of random variables, one for each firm as in Stevens (1998). Indeed, in the microfounded specification in Section 2, $\tilde{\boldsymbol{\mathcal{M}}}_D$ is proportional to the covariance matrix of the firm's cash flows and $\tilde{\boldsymbol{\mathcal{M}}}_S$ is proportional to a matrix that controls the relationship between firms' investment costs (and could have many of the properties of covariance matrices).

Interpreting $\tilde{\boldsymbol{\mathcal{M}}}_T$ in this way, the demand shift $\Delta \boldsymbol{K}$ associated with $\Delta K_i(n)$ can be viewed as the optimal "portfolio" of demand shifts adopted by the market given "covariance" matrix $\tilde{\boldsymbol{\mathcal{M}}}_T$ and "excess expected return" vector $\tilde{\boldsymbol{\mathcal{M}}}_D(\cdot, n)\Delta K_i(n)$. The denominator in $\mathcal{C}^I_{Cross}(n)$ then reflects the fact that the more the "portfolio covariance" (and investment costs) can be reduced by marginal offsetting positions in other assets, the more can be invested in the firm in question. The numerator in $\mathcal{C}^I_{Cross}(n)$ reflects the cost of implementing these "hedge" positions. Similarly, the enterprise contribution multiplier reflects the cost of these positions in terms of production of the non-market good. A firm with a positive but mediocre $g_i(n)$ could to have a negative enterprise contribution multiplier if it competes with firms with high values of g_i (i.e. if its $\beta_{T,n}$ is large for such firms). Equally, if a firm has positive values of β_T associated with firms with negative g, or negative values of β_T associated with firms with positive g, then its enterprise contribution multiplier could be above one.

1.2 Optimal demand curves and impact returns

In the previous subsection, I showed that marginal investors can have investor impact and that this impact depends non-trivially on the contribution multiplier matrix. In this subsection, building on the possibility of investor impact, I derive the optimal policy of an investor with a taste for the non-market good.

I assume that investor i's expected utility can be decomposed into:

$$U_i = U_{i,0} + U_i^H(\boldsymbol{K_i}) + U_i^G(\boldsymbol{K}) \tag{7}$$

with baseline financial utility $U_{i,0} = U_i^F - (1+r_f)\boldsymbol{P}'\boldsymbol{K_i}$, where U_i^F does not depend explicitly on $\boldsymbol{P}$. $U_i^H(\boldsymbol{K_i}) = \boldsymbol{h}_i'\boldsymbol{K}_i$, is a holdings-based preference term and $\boldsymbol{h}_i$ is a vector of exogenous holdings-based tastes that may be due to non-financial risk, subjective financial beliefs, and holdings-based ESG preferences (e.g. warm-glow). The impact preference term is $U_i^G(\boldsymbol{K}) = \gamma_i^g \boldsymbol{g}_i' \boldsymbol{K}$. The investor's "price of impact', γ_i^g, defines the strength of their taste for the non-market good.

In a competitive equilibrium no agent should be able to improve their utility by modifying their demand or supply curves, subject to the market clearing condition with the supply and demand curves of the other agents held fixed. In my baseline case, I assume that each firm's parameters are fixed so that the only endogenous variables are the $\boldsymbol{K}_i$, $\boldsymbol{P}$, and $\boldsymbol{K}$ and derive the following proposition in Appendix B .

Proposition 1.3. *Optimal investor demand curves*. *Suppose that $\boldsymbol{P}_{i,0}$ defines an investor's optimal demand curves when $\gamma_i^g = 0$ and $\boldsymbol{h}_i = 0$ (i.e. with only traditional financial preferences). Then for non-zero values of γ_i^g and $\boldsymbol{h}_i$ the optimal demand curves are given by*

$$\boldsymbol{P} = \boldsymbol{P}_{i,0}(\boldsymbol{K}_i, \boldsymbol{K}, \boldsymbol{X}) + \frac{1}{1 + r_f}\left(\boldsymbol{h}_i + \gamma_i^g \boldsymbol{\mathcal{C}}' \boldsymbol{g}_i\right). \tag{8}$$

This result shows that both holdings-based and impact preferences lead to strategic shifts in the investor's demand curves and that impact preferences are equivalent to holdings-based preferences with $\boldsymbol{h}_i = \gamma_i^g \boldsymbol{\mathcal{C}}' \boldsymbol{g}_i$. This means that all results for holdings-based preferences in the literature hold for impact preferences as well, as long as they are adjusted for the contribution multiplier matrix. This also means that impact preferences can generate an economically significant shift in investor demand for given sufficiently positive values for the price of impact (γ_i^g), the social productivities ($\boldsymbol{g}_i$), and the contribution multiplier matrix. In the empirical section of this paper I examine the evidence for whether or not this is the case for popular ESG metrics in the U.S. stock market.

Even if impact preferences produce economically significant portfolio tilts, the contribution multiplier matrix can be expected to have values substantially lower than 1 given the standard assumption that supply elasticities are much lower than demand elasticities. Note that the contribution multiplier matrix arises under a first order approximation and should only be expected to be valid for investors that are small relative to the relevant market. In particular, for a representative investor the contribution multiplier matrix will be the identity matrix. Thus, if all investors (or a substantial majority) were able to agree on their nonpecuniary preferences and coordinate their investor activities, then investor contribution would not be a concern. Such coordinated investors would likely invest substantially more in firms with high social productivity than uncoordinated investors. If there was broad agreement on nonpecuniary values then this would suggest a "price of anarchy" to investor's lack of coordination. However, common sense suggests that nonpecuniary preferences are highly heterogeneous and the heterogeneity observed in Koijen et al. (2022) underlines empirically how far investors are from full coordination. It thus could well be that if there is a price being paid, it is being paid by investors with strong nonpecuniary preferences who assume it is possible to coordinate with the market at large.

To understand what investors are willing to pay to optimize for their preferences, consider that Proposition 1.3 suggests that the investor acts as if they expect pseudo-returns

$$\boldsymbol{IR}_i = \gamma_i^g \operatorname{diag}(\boldsymbol{P})^{-1} \boldsymbol{\mathcal{C}}' \boldsymbol{g}_i \tag{9}$$

which I refer to as "**impact returns**". The impact return for firm n is then

$$IR_i(n) \equiv \frac{\gamma_i^g g(n) \mathcal{C}^I(n) \mathcal{C}^E(n)}{P(n)}, \tag{10}$$

and the associated "impact Sharpe ratio" is

$$\lambda_i^g(n) \equiv \frac{IR_i(n)}{\sigma_i(n)} \tag{11}$$

where $\sigma_i(n)$ is the investor's assessment of the volatility of firm n's financial returns.

These definitions of impact returns and impact Sharpe ratios are key to my empirical analysis. If impact returns can be observed, along with estimates of the contribution multipliers, then this can inform an estimate of investors prices of impact γ_i^g. The assessed prices of impact can inform discussion of whether investors are following holdings-based or impact preferences.

The shift in the demand curve from turning on the investor's impact returns shifts the investor's portfolio by their "impact tilt"

$$\Delta \boldsymbol{K}_i^{II} = \gamma_i^g \tilde{\boldsymbol{\zeta}}_i \boldsymbol{\mathcal{C}}' \boldsymbol{g}_i. \tag{12}$$

which generates an investor impact of $II_i^* = \gamma_i^g \boldsymbol{g}_i' \boldsymbol{\mathcal{C}} \tilde{\boldsymbol{\zeta}}_i \boldsymbol{\mathcal{C}}' \boldsymbol{g}_i$.

What should the impact-tilt portfolio be expected to look like? Well, heavy-tailed distributions of $\boldsymbol{g}_i$ are a stylized fact in the cost-benefit literature (see, for example, Gillingham and Stock (2018), Angrist et al. (2020), and Laxminarayan et al. (2006)). Additionally, as I show in Section 4.1, the values in $\boldsymbol{\mathcal{C}}$ are likely to vary over orders of magnitude in the cross section. These stylized facts suggest that the distribution of impact returns will have a highly skewed distribution. This implies that investors' impact tilts are likely to be concentrated on a few firms with exceptional impact returns.

Furthermore, investors subject to fixed transaction costs can be expected to prioritize investments in proportion to the level of their utility contribution: $\Delta U_i \propto \gamma_i^g II_i^* = (\gamma_i^g)^2 \boldsymbol{g}_i' \boldsymbol{\mathcal{C}} \boldsymbol{\zeta}_i \boldsymbol{\mathcal{C}}' \boldsymbol{g}_i$. Because the parameters in this expression are all squared, its distribution across firms can be expected to be more skewed than than the distribution of impact returns themselves. Hence, impact investors with high, fixed transaction costs are likely to have even more selective impact tilts.

Note that these points do not imply that investors with impact preferences will purely hold "impact investments". This could be the case for sufficiently high γ_i^g. However, for more moderate values of γ_i^g, what seems more likely is that the handful of firms with economically significant impact returns will be relatively over (or under) weight in an investor's portfolio, but not be the only holdings. In Section 3, I discuss how γ_i^g might be determined.

1.3 Accounting for passive investors

In the literature, passive investors are usually assumed to have fixed holdings and $\tilde{\boldsymbol{\zeta}}_D = 0$. However, as highlighted by Betermier et al. (2022) in an extension of their model, this need not be the case when supply is elastic. If shifts in demand by active investors cause supply to change, and passive investors react to these changes (i.e. to continue holding the market portfolio), then it is possible that the effects of active investors may be leveraged.

To account for this I assume that the set of investors may be partitioned into a subset of active investors, A, and a subset of passive investors, P. Each passive investor seeks to hold

$\boldsymbol{K}_P = \boldsymbol{\kappa}_P \boldsymbol{K}_A$, with $\boldsymbol{\kappa}_P$ a diagonal matrix that summarizes the multiple of the active portfolio they choose to hold for each stock. These multiples will depend on a mix of, for example, the relative wealth and risk aversion of the passive investors as in Betermier et al. (2022), the inelastic demand structure explored in Gabaix and Koijen (2021), or the benchmarking intensity of Pavlova and Sikorskaya (2023).

These assumptions imply that $\boldsymbol{K}_D = (\boldsymbol{I}+\boldsymbol{\kappa}_P)\boldsymbol{K}_A$ and thus that $\tilde{\boldsymbol{\zeta}}_D = (\boldsymbol{I}+\boldsymbol{\kappa}_P)\tilde{\boldsymbol{\zeta}}_A$, where $\tilde{\boldsymbol{\zeta}}_A$ is the aggregate demand elasticity matrix of the active investors. This means that for an active investor i, $\frac{\partial \boldsymbol{P}}{\partial \boldsymbol{K}_i} = (\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1}(\boldsymbol{I}+\boldsymbol{\kappa}_P)$ and hence $\frac{\partial \boldsymbol{K}}{\partial \boldsymbol{K}_i} = \tilde{\boldsymbol{\zeta}}_S(\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1}(\boldsymbol{I}+\boldsymbol{\kappa}_P)$. The contribution multiplier matrix is thus given by

$$\boldsymbol{\mathcal{C}} \equiv \tilde{\boldsymbol{\zeta}}_S(\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1}(\boldsymbol{I}+\boldsymbol{\kappa}_P). \tag{13}$$

2 Microfounded case

To illustrate the implications of the model in an intuitive setting, in this section I solve the model based on microfounded utility functions for both the firms and investors.

Suppose each investor has standard mean-variance utility plus a taste for the non-market good as in the previous sections,

$$U_i = E[W_{i,1}] - \frac{\gamma_i}{2W_{i,0}} Var[W_{i,1}] + \gamma_i^g G, \tag{14}$$

where γ_i defines the investor's level of financial risk aversion.

At time 1, each firm generates free cash flow $CF(n) = z_{CF}(n)K(n)$. $\boldsymbol{z}_{CF}$ is a random variable with exogenous mean vector $\boldsymbol{a}_{CF}$ and covariance $\boldsymbol{\Sigma}_{CF}$. To keep the model self-contained within the single period, I assume that the cash flows captures not just profit from production during the period but also the ongoing market value of the installed capital.

As in Section 1.3, I assume a split between active and passive investors with passive investors holding portfolio $\boldsymbol{K}_P = \boldsymbol{\kappa}_P \boldsymbol{K}_A$. Furthermore, as in Merton (1987) and Koijen et al. (2022), each active investor allocates $W_{i,0}$ at time 0 across assets in its opportunity set $\mathcal{N}_i \subseteq \{1, ..., N\}$ and the riskless asset. The opportunity set may be a subset of the entire investment universe for a variety of underlying reasons, including lack of information, limitations on short sales, taxes, transaction costs, and other frictions, as well as divestment commitments. I denote the number of assets in the investor's opportunity set as $|\mathcal{N}_i|$. Let $\boldsymbol{I}_i$ be an $|\mathcal{N}_i| \times N$ matrix where for each $n \in \mathcal{N}_i$ the corresponding row in $\boldsymbol{I}_i$ is $\boldsymbol{e}_n'$. These matrices will be used to determine the aggregate demand curves formed by investors with different opportunity sets. They can also be used to impose short-sale constraints (by excluding for an opportunity set firms an investor would otherwise short).

Each firm's technology exhibits decreasing returns to scale, both in regards to the firm itself and its competitors. As in Zhang (2017), I capture this as an adjustment cost that increases the amount of investment necessary to install $\Delta K(n)$ of new capital. In addition to a basic cost of $\Delta K(n)$ to install the new capital, I assume that the adjustment costs are

$$\Phi(n) = \frac{1}{2}\Sigma_C(n,n)\Delta K(n)^2 + \sum_{m \neq n} \Sigma_C(n,m)\Delta K(n)\Delta K(m), \tag{15}$$

where $\boldsymbol{\Sigma_C}$ is a matrix of cost sensitivities. These sensitivities could arise due to competition for resources within an industry or externalities between firms in different industries. As in Zhang (2017), it could be that $\Sigma_C(n,n) \propto K_0(n)^{-1}$ so firms have economies of scale with respect to already installed capital. This part of the model can also be used to capture decreasing returns to scale, both for firms and funds (as in Berk and Green (2004)). For simplicity, I assume in this paper that $\boldsymbol{\Sigma_C}$ is symmetric and invertible. This assumption makes sense, for example, if adjustment costs depend on the total new capital set to be installed in an industry, independent of how much each individual firm is installing. In general, though, there could be asymmetric effects.

I assume that each firm is managed on behalf of existing shareholders who consume the economic profit—the difference between the firm's market value and capital installation costs. The utility of firm n is thus:

$$U_F(n) = P(n)K(n) - \Delta K(n) - \Phi(n). \tag{16}$$

As in my baseline model, I assume firms' social productivities are exogenously specified. I do this because I am uncertain about how to microfound the parameters that are central to the model for endogenous social productivities that I examine in Section 5, in particular the cost of change.

For the above specification I derive the following proposition in Appendix C.

Proposition 2.1. ***Optimal microfounded supply and demand curves****. To first order, the optimal demand curves are*

$$\boldsymbol{P} = \frac{(\boldsymbol{a}_i + \gamma_i^g \boldsymbol{\mathcal{C}}' \boldsymbol{g}_i)}{1 + r_f} - \boldsymbol{\zeta}_i^{-1} \boldsymbol{K}_i, \tag{17}$$

and the optimal supply curves are

$$\boldsymbol{P} = \mathbf{1} + \boldsymbol{\zeta}_S^{-1}(\boldsymbol{K} - \boldsymbol{K}_0), \tag{18}$$

with supply and demand elasticity matrices

$$\boldsymbol{\zeta}_i = (1 + r_f)\frac{W_{i,0}}{\gamma_i}\boldsymbol{\Sigma}_{CF,i}^{-1}, \tag{19}$$

$$\boldsymbol{\zeta}_D = (\boldsymbol{I} + \boldsymbol{\kappa}_P)\left(\sum_{i \in A}^{I} \boldsymbol{I}_i' \boldsymbol{\zeta}_i \boldsymbol{I}_i\right), \tag{20}$$

$$\boldsymbol{\zeta}_S = \boldsymbol{\Sigma}_C^{-1}. \tag{21}$$

The supply elasticity matrix is determined by the firm cost-sensitivity matrix and firms with higher adjustment costs will be less elastic and have lower contribution multipliers. Firms with more volatile cash flows or that are in fewer investors' opportunity sets will tend to have lower demand elasticities and higher contribution multipliers.

Suppose the investor tilts their portfolio not necessarily by the optimal impact tilt but by $\boldsymbol{\mu}_i^g$. The relevant part of their utility function is

$$U_i^{(g)} = \frac{W_{i,0}}{\gamma_i}\left(\gamma_i^g \boldsymbol{g}_i' \boldsymbol{\mathcal{C}} \boldsymbol{\Sigma}_{CF,i}^{-1} \boldsymbol{\mu}_i^g - \frac{1}{2}(1 + r_f)(\boldsymbol{\mu}_i^g)' \boldsymbol{\Sigma}_{CF,i}^{-1} \boldsymbol{\mu}_i^g\right). \tag{22}$$

This utility term is indeed maximized with the optimal demand curves and is equal to $U_i^{(g)} = \frac{1}{2}\frac{W_{i,0}}{(1+r_f)\gamma_i}(\gamma_i^g)^2 \boldsymbol{g}_i' \boldsymbol{\mathcal{C}} \boldsymbol{\Sigma}_{CF,i}^{-1} \boldsymbol{\mathcal{C}}' \boldsymbol{g}_i$. Note that, heuristically, $U_i^{(g)}$ will be negative if the investor's impact tilt is more than a factor of two greater than the optimal shift (or of the opposite sign). Thus, for investors with $\gamma_i^g > 0$, it may be much worse to overestimate the appropriate demand shift than to underestimate it (but still being of the correct sign).

2.1 Relationship between impact and financial returns

The relationship between expected financial returns and impact returns is a topic of perennial debate and mixed results (Starks, 2023). It is important because investors with different beliefs about this may strategically allocate their time in ways that will result in different portfolios. An investor that expects to see a positive correlation between impact and financial returns is likely to search for and hold a portfolio of firms with both significantly positive a_i and g_i. Whereas an investor that believes in a negative correlation will be likely to hold a mix of 'non-sustainable' firms for their superior financial returns and concessionary impact investments for their superior impact returns. These different types of investors are likely to devote their search to different parts of the overall opportunity set.

For the specification in this section, the expected financial returns are $\boldsymbol{FR}_i = \text{diag}(\boldsymbol{P})^{-1}\boldsymbol{a}_i - 1$ and the impact returns are $\boldsymbol{IR}_i = \gamma_i^g \, \text{diag}(\boldsymbol{P})^{-1}\boldsymbol{\mathcal{C}}'\boldsymbol{g}_i$. Independent of any relationship between the a_i and the g_i (which I don't comment on here), these returns may be correlated due to their shared dependence on prices and hence the supply and demand elasticities. In Appendix C.3, I show that equilibrium prices are (for simplicity, ignoring the effect of the initial firm sizes):

$$\boldsymbol{P} \approx \frac{(\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1}\tilde{\boldsymbol{\zeta}}_D}{1 + r_f}(\boldsymbol{a} + \gamma^g \boldsymbol{\mathcal{C}}'\boldsymbol{g}). \tag{23}$$

In this simple case, an increase in the demand elasticity associated with a firm can thus be expected to both increase prices and decrease the relevant contribution multiplier. This can be expected to decrease both financial and impact returns. In contrast, an increase in the supply elasticity associated with a firm can be expected to decrease prices and increase the relevant contribution multiplier. This can be expected to increase both financial and impact returns. Overall, this indicates that the supply and demand system generates a positive relationship between financial and impact returns. Taken together this connection between expected financial and impact returns, plus the expression for the impact tilt in equation (12), suggests the image of an ideal impact investment is a large, positive position in a firm that is highly profitable, highly socially productive and with highly elastic supply yet inelastic demand. Cole et al. (2020) provides an empirical example of arguably exactly this relationship being realized in the portfolio of the IFC.

In other words, if an investor can position themselves to “have their cake and eat it too” then there is no reason not to. Of course, in practice it could be that all highly profitable firms attract significant levels of elastic demand, making the ideal impossible to find. Nevertheless, given fixed elasticities, impact investors should prefer more profitable firms. And given fixed profitability and social productivities, investors should prefer firms with less elastic demand.

3 The price of impact

A key part of my model is that each investor’s “price of impact”, γ_i^g, determines their taste for the non-market good and hence their optimal demand shift. In this section I discuss how the cost-effectiveness of charitable opportunities, and other types of opportunities, can be used to obtain upper bounds on an investor’s price of impact.

An investor’s price of impact is not a normal “price” in that it is a subjective parameter and not based on any market for buying and selling “impact” (which doesn’t exist in the model and may not be possible to create in practice). In this paper I present the price of impact as exogenous. But, in general one can imagine it being set strategically by sophisticated investors based on a combination of their beliefs about current and future opportunities as well as other preferences.

For example, if opportunities at future times are forecast to offer both abnormally high social productivities or contribution multipliers, then setting a low value for γ_i^g in the current period, reducing their impact tilt and staying closer to their financially optimal portfolio, could maximize their expected utility over time.

One way to determine an investor’s price of impact is based on the revealed preferences. Given assessments of social productivities and the contribution multipliers, the price of impact can be inferred from the impact returns implied by observed portfolio tilts. However, another way to infer this parameter is to consider investors’ choices regarding high-impact concessionary opportunities. In particular, charitable opportunities. I develop this simple idea in the rest of this section and put both approaches to determining the price of impact to work in Section 4. For other models that have accounted for the option many investors have to donate to charity, see for example Graff Zivin and Small (2005), Baron (2007), and Baron (2009).

If the investor is a private individual, the set of available charitable opportunities is vast. For an institutional investor, fiduciary duties and operating procedures may limit the available opportunities, though sponsoring research reports or shareholder engagement initiatives may be an option. For some fiduciaries (e.g. asset managers, pension funds) it may be appropriate, for this exercise, to consider the charitable opportunities available to their clients[4].

In any case, suppose that CE_i^* is investor i's assessment of the cost-effectiveness (cost per unit of the non-market good) of the best available charitable opportunity[5]. In equilibrium the investor will not be able to gain utility by donating more to this charity (or to less effective options). Assuming their marginal investment is in the riskless asset, this implies that

$$\gamma_i^g \leq (1 + r_f) CE_i^*, \tag{24}$$

is an upper bound on the price of impact. An investor who prices impact at a higher rate than this will be paying more for it than the available opportunities require (because of the existence of the charitable opportunity).

The price of impact need not be at this upper bound. That is, $\gamma_i^g < (1 + r_f) CE_i^*$ may hold. For example, an investor may choose to not have any impact tilt in the present time period. This could be due to a lack of altruistic preferences. Or, it could be because they forecast higher impact opportunities in the future and wish to save to fund these future opportunities.

More generally, for investors who do not fund explicitly charitable opportunities, one might consider a 'best available concessionary opportunity' with risk-adjusted expected financial return μ_i^*, social productivity g^*, and investor and enterprise contribution multipliers $\mathcal{C}_*^I$ and $\mathcal{C}_{i,*}^E$. The assumption that the investor can invest in this opportunity but chooses not on the margin implies that $\mu_i^* + \gamma_i^g \mathcal{C}_*^I \mathcal{C}_{i,*}^E g^* / P_* \leq (1 + r_f)$, which gives an upper bound for

[4]For example, the asset manager Fidelity Charitable made over $9 billion in donor-recommended grants in 2020 to 170,000 organization via donor-advised funds (https://www.fidelitycharitable.org/ insights/2021-giving-report.html). If at least some of this giving was informed by cost-effectiveness assessments, then it could be possible to assess the marginal cost-effectiveness of these donations.

[5]Available meaning a charity they can legally donate to and that still has need for more funding.

the price of impact of

$$\gamma_i^g \le \frac{P_*((1+r_f) - \mu_i^*)}{\mathcal{C}_*^I \mathcal{C}_{i,*}^E g^*}. \tag{25}$$

Note that if this upper bound is negative, it would suggest that the investor's marginal asset is not the riskless asset.

The price of impact γ_i^g in my model is different from and complementary to the use of social costs estimates in the large literature on cost-benefit analysis. Investors may use social costs, for example, to attempt to compare different non-market goods based on careful consideration of societal preferences. The price of impact, instead, is what the investor uses to optimize the strength of their impact tilt based on their opportunity set.

Another perspective on this relationship is that estimates of social costs are just another potential upper bound on γ_i^g. Indeed, in many cases it would be hard to justify spending more to avert an externality than society (i.e. the beneficiary) is itself willing to pay—though the investor could believe that social costs underestimated. Whether or not the relevant social cost upper bound is lower than the charitable bound is an empirical question. By and large, however, I would expect that an unconstrained investor should be able to find charitable opportunities that avert impact for less than the social cost.

Note that the differences between social costs and alternative bounds can be large. For example, the value of a statistical life in the US is assessed to be on the order of magnitude of \$10 million[6]. Whereas, the charity evaluator GiveWell consistently estimates that the charities they recommend save lives (in the developing world) at a rate of less than ten thousand dollars per life saved[7].

In some cases, the social cost may actually be more relevant to forecasting firm profitability than to setting the price of impact. For example, suppose a government has decided that the social costs of an externality are high. The government then passes policies that offer to pay for products that address the externality (e.g. electric vehicle subsidies). Further, suppose that an investor believes the government has overestimated the social costs of the externality, or is otherwise implementing overly generous policies. Then the investor is

[6]https://www.transportation.gov/office-policy/transportation-policy/revised-departmental-guidance-on-valuation-of-a-statistical-life-in-economic-analysis. Accessed: 2022-10-31.

[7]See, for example: https://www.givewell.org/cost-to-save-a-life. Accessed: 2022-10-31.

unlikely to want to set their private γ_i^g based on these policies, but they may see a valuable financial opportunity in allocating capital to a commercial response.

4 Empirically assessing investor impact returns

In this section, I combine results from Betermier et al. (2022), Koijen et al. (2022) and Merton (1987) to offer initial results on how investors are pricing impact and how investor contribution multipliers may vary across firms. Comprehensive empirical study, especially the estimation of heterogeneous supply elasticities, presents many challenges and I leave such investigations as an important avenue for future research.

4.1 Investor contribution multipliers

Homogeneous estimate. Betermier et al. (2022) estimate the supply and demand curves that apply to a panel of 1,226 large- and medium-sized U.S. firms over the period 1999-2019. They estimate the following econometric specification using three-stage least squares[8]:

$$\lambda_{n,t} = \boldsymbol{\beta}_D' \boldsymbol{x}_{D,n,t} + \Delta_D \rho_{M,n,t} + u_{D,n,t}, \tag{26}$$

$$\lambda_{n,t} = \boldsymbol{\beta}_S' \boldsymbol{x}_{S,n,t} + \Delta_S \rho_{M,n,t}^{int} + u_{S,n,t}, \tag{27}$$

where $\boldsymbol{x}_D$ and $\boldsymbol{x}_S$ are vectors of firm characteristics that are expected to drive supply and demand, $\rho_{M,n,t}$ is the correlation with the market portfolio, and $\rho_{M,n,t}^{int}$ is portion of $\rho_{M,n,t}$ estimated to come from a stock's correlation with itself (used as a measure of firm size). Homogeneous slopes Δ_D and Δ_S, and linear coefficients $\boldsymbol{\beta}_D$ and $\boldsymbol{\beta}_S$, are imposed across firms.

As reported in Table 1, the slope estimates by themselves imply an investor contribution multiplier of 0.9%. However, the actual investor contribution multiplier will depend on the wealth ratio between active and passive investors as in equation (13). Based on the analysis of active shares over time in Koijen et al. (2022), I use 35% as the active share for my analysis as this roughly corresponds to the active share during the middle of the sample periods of both Betermier et al. (2022) and Koijen et al. (2022). Combined with this active share

[8]Note that they frame their model in terms of the supply and demand for capital, not assets, so I have swapped the labeling of their equations to match the meaning of supply and demand in the present paper.

Demand slope (Δ_D)	0.666	
	(0.143)	
Supply slope (Δ_S)	−73.635	
	(42.129)	
Active share	1.00	0.35
Investor contribution multiplier (%)	0.9	2.6

Table 1: **Investor contribution multipliers based on homogeneous slope estimates**. This table reports estimates related to the investor contribution multiplier for the large and medium U.S. firms. The upper panel reports the slope estimates from Betermier et al. (2022). The lower panel reports the investor contribution multipliers that can be inferred from these slopes and either an active share of 1.00 or the active share based on Koijen et al. (2022). The slope estimates are for an unbalanced panel of 1,226 U.S. firms observed on December 31 of years 2004, 2009, and 2014.

estimate, the slope estimates reported in Table 1 imply an investor contribution multiplier of 2.6%. Note that this estimate ignores the more complex terms in equation 5, though I expect that accounting for these terms would more likely than not decrease the estimated contribution multiplier (increasing the tension between impact preferences and observed tilts that I find later in this section).

Potential variation in investor contribution. In Table 2, I extend an analysis of Merton (1987) to show how investor contribution might vary for firms with different associated supply and demand elasticities. Merton (1987) sorts the sample of 1,387 U.S. firms in his 1985 dataset into 10 groups with the same number of firms in each group. He then uses variation in market value per registered shareholder to form rough estimates of the relative size of the investor base of each group. That is, the ratio of the percentage of investors with a firm in their opportunity sets to the same percentage for firms in the largest group.

I set group 8 to have the demand slope from Table 1. The supply and demand slopes are then scaled appropriately across the rows and columns of Table 2. To cover firms that have even smaller investor bases than in Merton's sample (e.g. small private firms), I add two additional rows with investor bases 10 and 100 times lower than group 1. To capture a

Merton Group	% Total Market Value	Relative Size of Investor Base	AS	Relative Demand Elasticity	Relative Supply Elasticity .001	.01	.1	1	10
		0.0002	1	0.001	0.8	7.6	45.1	89.1	98.8
		0.002	1	0.01	0.1	0.8	7.6	45.1	89.1
1	0.1	0.02	1	0.1	0.0	0.1	0.8	7.6	45.1
2	0.3	0.03	1	0.17	0.0	0.1	0.6	5.8	39.3
3	0.7	0.04	0.6	0.22	0.0	0.1	0.5	4.9	36.4
4	1.2	0.05	0.6	0.28	0.0	0.1	0.5	4.9	38.0
5	2	0.06	0.6	0.33	0.0	0.0	0.5	4.8	39.1
6	3.2	0.09	0.35	0.5	0.0	0.0	0.4	4.0	34.3
7	5.3	0.13	0.35	0.7	0.0	0.0	0.3	3.1	28.0
8	8.7	0.18	0.35	1.0	0.0	0.0	0.3	2.6	23.9
9	15.1	0.34	0.35	1.9	0.0	0.0	0.1	1.5	14.2
10	63.5	1	0.35	5.6	0.0	0.0	0.0	0.5	5.4

Table 2: **Potential variation in investor contribution multipliers**. This table reports illustrative calculations of investor contribution for firms with different associated demand elasticities ($\tilde{\zeta}_D$) and supply elasticities ($\tilde{\zeta}_S$). The ten rows of numbered groups, and the '% Total Market Value' and 'Relative Size of Investor Base' columns are based on Table I in Merton (1987). The relative size of the investor base is an estimate of the proportion of investors that have the average stock from a group in their opportunity sets. 'AS' is the active share assumed for each group. The investor contribution multipliers are reported as percentages. The table is calibrated so that the cell with relative supply and demand elasticities of 1 has the investor contribution given in Table 1. Cells with investor contributions less than this reference value are shaded gray.

range of supply elasticity values, I include columns with supply elasticities ranging from 1000 times less than the corresponding estimate in Table 1 to 10 times higher. To illustrate the likely possibility that firms with smaller investor bases have much less passive investment, I vary the active share from 0.3 for the largest group to 1 for group 1.

The results illustrate two main points. First, investor contribution multipliers can vary by multiple orders of magnitude depending on the supply and demand elasticities. Second, the variation is not all due to the demand elasticity. A firm that is associated with relatively inelastic demand but also a lower supply elasticity, and which is only covered by active investors, can have an investor contribution that is significantly lower than that of much larger firms that are associated with more elastic demand. Investors that are systematically able to identify firms with high supply elasticities can potentially generate relatively large investor contributions, even if these firms are not the most neglected in terms of the demand elasticity of their investor base. Similar to the conclusion of Blitz and Swinkels (2020), investors may wish to differentiate their policies depending on the assessed supply and demand elasticities of individual firms, which are likely to vary both in the cross-section and over time.

The research implication of these results is that if investors with impact preferences are accounting for variations in the relevant investor contribution multipliers, both in the cross-section and over time, then this could have a large effect on estimation results.

4.2 Impact returns and prices of impact

Impact returns. In the first part of the analysis in this subsection, I convert empirical estimates of the demand shifts associated with popular ESG metrics into impact returns. These results are summarized in Table 3.

Social score. As part of their study, Betermier et al. (2022) estimate that a one-standard deviation improvement in a firm's MSCI 'Social' (S) score reduces its equilibrium Sharpe ratio by 0.025. For the average firm in their cross-section, with a return volatility of 36%, such a decrease corresponds to a 0.9 percentage point absolute decrease in expected return. Their findings suggest that changes in nonpecuniary characteristics can indeed have an economically significant impact on firms.

Given the heterogeneity observed in Koijen et al. (2022), for the overall market to price in an impact return of $IR^{1SD} = 0.9\%$, it must be the case that some investors apply much higher impact returns to the same assets. To illustrate this in a way that is consistent across the metrics that I examine in this section, I also report results for demand coefficients that are five times larger. Such larger coefficients are well within the ranges reported by Koijen et al. (2022) and Noh et al. (2023).

Environmental score. Using an approach focused on leveraging investor holdings data, but using a reasonably similar U.S. data set to the above estimates, Koijen et al. (2022) estimate the shift in investor demand curves from a one standard deviation increase in firms' Sustainalytics 'environmental' (E) score. They report estimated demand coefficients for each institution in their sample, with the estimates ranging from a roughly -40% decline in demand to a 25% increase in demand (i.e. percentage increase in the target weight in the institution's portfolio). As a representative value I use the average coefficient of roughly 5% for households (the largest type of investor over the sample)[9].

I convert this estimate into an implied impact return and price of impact for the average firm in the cross-section of Betermier et al. (2022). Based on the Sharpe ratio of this average firm, a 5% increase in demand corresponds to a 0.024 change in Sharpe ratio and a 0.9% shift in return. This is remarkably similar to the respective values for the social score.

Emissions intensity and adjusted environmental score. Noh et al. (2023) follow an approach similar to Koijen et al. (2022), but with a dataset of US stocks from 2013 to 2021 and with both an environmental score and a greenhouse gas (GHG) emissions intensity score. Their log-emissions intensity is the logarithm of firms' Scope 1 GHG emissions to revenue ratios. The log-emissions intensity has an average cross-sectional correlation of 58% with the MSCI Environmental Score, so they define their environmental score as the residuals of the MSCI environmental score after regression on the log-emissions intensity. I convert the estimated demand coefficients for these scores into impact returns using the same procedure as for Koijen et al. (2022)'s environmental score.

Note that Betermier et al. (2022)'s estimation takes place on five-year windows while Koijen et al. (2022) and Noh et al. (2023) use quarterly data. Intuitively this would be

[9]This is a rough estimate as this result is only reported in a histogram chart, not a table. Passive investors, long-term investors and brokers have lower average demand coefficients, but of the same sign and order of magnitude, ranging from 1% to 4%. Hedge funds, small active investors and private banking have negative coefficients, ranging from -1% to -4%. They also report a regression which finds a 17% increase in market-to-book ratio for a one standard deviation increase in the E-score. However, these results are from a preliminary regression without the careful use of instruments they apply to generate their main results.

more likely to be an issue if I were comparing price elasticity estimates from these different horizons as both supply and demand are likely to appear more elastic at longer horizons (Gabaix and Koijen, 2021; Van der Beck, 2022). However, it is not clear that there should be a similar horizon-dependence problem for impact returns and the similar results from the different sources in Table 3 offers some comfort in this regard.

Prices of impact. Next, I infer the prices of impact associated with the impact returns reported in Table 3. For compatibility with how I treat the emissions intensities (which are in tons of GHG emissions per million dollars revenue), I assume that the non-market good relevant to the ESG scores is the sum of firm's revenues multiplied by their respective ESG scores. The price of impact for a one standard deviation improvement, γ^{1SD} is then the investor willingness to pay to improve the ESG score associated with one dollar of revenue.

To calculate the γ^{1SD} prices of impact, I first divide the impact returns by the larger investor contribution multiplier from Table 1. I then multiply by a market value to revenue ratio of 2, which I take to be the average for the relevant market[10]. This reflects that the impact returns are relative to market values, while as just discussed I assume that the investors' real concern is the operating size of the firms in terms of their revenues.

Finally, for the log-emissions intensity I convert the one standard deviation prices of impact into prices of impact per tonne of GHG emissions averted. The choice of the relevant conversion factor is not straightforward because the metric is the logarithm of the intensity not the actual intensity. To be conservative in the direction of a lower GHG price of impact, I assume that the relevant standard deviation factor is equivalent to 1,000 tons of GHG emissions per million dollars revenue. The standard deviation of the Scope 1 GHG-emissions intensities reported in Bolton and Kacperczyk (2021) and Noh et al. (2023) are near 600 (with much lower standard deviations are much lower for Scopes 2 and 3). I divide the relevant γ^{1SD} prices by this factor to arrive at the γ^{GHG} prices of impact.

Bottom-up estimates for GHG emissions. As discussed in Section 3, it is also possible to form a bottom-up estimate of impact returns based on prices of impact informed by charitable cost-effectiveness estimates and social costs.

What price of impact might investors choose for GHG emissions? Carbon markets and

[10]One point of evidence for this ratio on a relevant population is that Fossil Free Funds report that the average GHG emissions intensity per dollar invested of the Russell 1000 is 94, while the revenue intensity is 181. These Russell 1000 carbon footprint values are from https://fossilfreefunds.org/fund/ishares-russell-1000-etf/IWB/carbon-footprint/FSUSA00B5K/FEUSA0001A. Accessed on 2022-10-31.

				Prices of impact	
Source	Metric	Original estimate	IR^{1SD}	γ^{1SD}	γ^{GHG}
BCJ	S score	0.025	0.9%	0.71	
BCJ	S score	0.125	4.5%	3.53	
KRY	E score	0.05	0.9%	0.67	
KRY	E score	0.25	4.3%	3.33	
NOS	log GHG	-0.023	−0.4%	0.31	306
NOS	log GHG	-0.115	−2.0%	1.53	1,530
NOS	Residual E	0.031	0.5%	0.41	
NOS	Residual E	0.155	2.6%	2.06	
	GHG		−0.003%	0.001	1
	GHG		−0.3%	0.1	100

Table 3: **Impact returns and prices of impact**. This table reports estimates of the impact returns and prices of impact associated with the findings of Betermier et al. (2022) (BCJ), Koijen et al. (2022) (KRY), and Noh et al. (2023) (NOS), and bottom-up estimates for GHG emissions. The impact returns are all for a one standard deviation increase in the relevant metric. The impact returns associated with the sourced estimates are shown both for the original average coefficient from the relevant source and a value five times larger to illustrate the heterogeneity of investor demand. The prices of impact are the price to improve the social productivity of one dollar of revenue by one standard deviation and to avert 1 ton of GHG emissions, respectively. The GHG estimates are shown for low and high prices of impact, based on published social costs and charitable cost-effectiveness estimates.

government policies have typically priced emissions at between \$10-100 per ton of GHG emissions in recent years. Rennert et al. (2022) suggests the social costs should be set at \$185 based on their models. However, in the context of optimal policy for the marginal investor, it is important that the world's largest governments have set much lower prices than this and some prominent charity evaluators argue targeted donations can achieve cost-effectiveness better than \$1 per tCO2e[11]. Therefore, I use \$1 per tCO2e and \$100 per tCO2e to define a range of plausible prices of impact based on observed charitable and social costs.

For a standard deviation of 1,000 tons of GHG emissions per million dollars revenue, a price of \$100 per ton implies $\gamma^{1SD} = 0.1$, whereas \$1 per ton implies $\gamma^{1SD} = 0.001$. I then convert these prices into implied impact returns using the investor contribution multiplier from Table 1. The results are summarized in Table 3.

Discussion. The prices of impact reported in Table 3 suggest that for the observed tilts to be consistent with impact preferences, some investors would need to be willing to pay substantial fractions or more of a dollar in order to improve the ESG score associated with one dollar of revenue. This is implies an implausible level of latent investor commitment to subsidizing more sustainable firms. The observed prices of impact are also all much larger than the bottom up prices of impact associated with GHG emissions based on observed estimates of social costs and charitable cost-effectiveness. I discuss the implications of this tension in the results in the next subsection.

Berk and Van Binsbergen (2021) argue that the impact of divestment on firms' cost of capital is limited. My results here are in line with their findings in two ways, but nevertheless highlight that divestment could be part of investors strategies given a sufficiently high price of impact. First, for an investor to divest (but not sell short) a firm in my model requires that their impact tilt offset the small risk premium associated with the firm's correlation with itself as part of the market portfolio (the "internal market correlation" in the terminology of Betermier et al. (2022)). The sample average internal market correlation reported by Betermier et al. (2022) is only 15 basis points. Assuming short selling constraints, this small value is the lowest impact return an investor can meaningfully assign to an (average) divested firm—whereas positive investments in high impact firms can be generated by impact returns with no upper bound. Second, the investor contribution multiplier reported in Table 1 means that investors who do divest will have an even smaller effect on the underlying

[11]See, for example, https://founderspledge.com/stories/climate-and-lifestyle-report.

firm's size than the already small impact returns associated with divestment (i.e. 15 bps) would suggest. However, 15 basis points is in the middle of the range for the bottom-up GHG impact returns, IR^{1SD}, reported in Table 3. This means that while investors who use a lower value of the price of impact for GHG emissions, γ^{GHG}, will merely slightly underweight high-GHG intensity firms, investors who use a higher price of impact for GHG emissions may still coherently divest, despite the limited investor impact.

4.3 Discussion

The results of the previous section present a clear tension between the magnitude of prices of impact inferred from observed ESG tilts and prices of impact estimated based on actual firm metrics and known social and charitable costs. There are many possible resolutions to this tension which I now discuss.

First, the investors in the relevant data may simply not have a taste for investor impact in the way I have defined it in this paper. In line with this possibility, Heeb et al. (2022) find that investors have a substantial willingness to pay for climate impact, but that they are not scope sensitive. Such investors may have a taste for impact that is not about their investor impact, but rather about their reputation or other qualitative benefits that depend on their holdings, and thus that doesn't depend on their investor contribution. Indeed, ESG ratings, by boiling complex quantitative criteria that vary dramatically across firms into simple linear scores, may be catering (or contributing) to such scale-insensitive tastes.

For other investors, such as professional impact activists, it could be that their "theory of change" is not well captured by my model (which is focused on the impact of capital allocation) or by simple ESG scores. They might have an explicit model of their impact that finds higher potential in firms that appear more susceptible to activist campaigns, that have ESG ratings momentum, or that are near a subjective threshold between good and bad ESG performance. It could also be that many firm's production of the non-market good are not well described by the simple linear function I have used ($G = gK$). For example, a firm's marginal project could be much dirtier than its average project, so reducing its size at the margin may be much more impactful than my baseline model would indicate.

Second, investors could believe that higher ESG scores are predictive of future profitability or other financial benefits (such as lower cash flow volatility). This would remove the role for the contribution term, thereby dissolving the tension. However, attempts to advocate for this resolution would need to square this result with the increasing number of papers that find investors are willing to pay for impact, including Heeb et al. (2022) and Barber et al. (2021).

Third, investors could have a taste for investor impact as I have defined it, but simply see a lot more value in ESG scores than the CO2 intensities to which I have compared them, or believe that the contribution multiplier that applies to their investments is much higher than the one I estimated based on Betermier et al. (2022)'s empirical results .

Fourth, it could be that the empirical results I have used as a basis for my analysis are somehow inappropriate for the role I have given them. That is, if investor contribution multipliers are positively correlated with the impact return applied by the market to each firm, then the estimates of Betermier et al. (2022) may be valid on their own terms, but biased when it comes to estimating the (investor-contribution adjusted) price of impact.

Note that there are factors I have not included in this analysis that could widen the gap between observed and 'bottom-up' impact returns. First, I have not adjusted for enterprise contribution multipliers or the more complex terms in equation 5. I expect these terms would result in a small downwards adjustment to the contribution multiplier for an average firm, which would increase my estimated prices of impact for the E and S scores and decrease my 'bottom-up' climate impact return estimates. Second, my analysis assumes the average non-ESG investor is neutral and does not account for the possibility that some investors adopt the opposite views of ESG investors, either as a financial strategy or for other reasons (i.e. anti-ESG). If a significant number of investors did this, this would increase the degree to which ESG investors need to tilt towards higher scoring stocks in order to produce the observed market tilt. Koijen et al. (2022) provides evidence that indeed more active and sophisticated investors are more likely to have negative demand coefficients associated with ESG scores.

In general, treating supply and demand in an integrated way has important implications for empirical work. At longer horizons, it could be that cases of high real impact due to elastic supply also feature little price impact, while situations of low real impact due to inelastic supply feature the price impacts that have been the target of most empirical work.

5 Extensions

In this section I consider extensions to my baseline setting to allow firms to endogenously modify their social productivities and to allow for a search model of investment.

5.1 Endogenous firm social productivities

Firm parameters are fixed in my baseline model. As in Pástor et al. (2021), I now allow firm n's manager to choose $g(n)$ in addition to $K(n)$. Each firm has initial social productivity $g_0(n)$. The manager chooses $\Delta g(n)$ which moves the firm's total social productivity to $g(n) = g_0(n) + \Delta g(n)$ but with an cost of $-\frac{\chi(n)}{2}(\Delta g(n))^2$ for each unit of installed capital $K(n)$, where $\chi(n) > 0$ controls the adjustment costs.

For simplicity, in this subsection I focus on the partial equilibrium for a single firm, ignoring cross-elasticities, and I assume all agents agree on the value of $g(n)$. For this case I derive the following proposition in Appendix C.3.

Proposition 5.1. ***Optimal demand shift with endogenous social productivity****. The optimal demand curve shift with endogenous social productivities is approximately*

$$\gamma_i^g \left(\mathcal{C}(n,n)\Big(1 + \frac{\Delta g(n)}{g(n)}\Big) g(n) + \frac{\zeta_D^{-1}(n,n)K(n)}{\chi(n)g(n)} \right). \tag{28}$$

For the microfounded case of Section 2, this shift further simplifies to

$$\gamma_i^g \mathcal{C}(n,n)\Big(1 + \frac{\Delta g(n)}{g(n)} + \frac{\mu(n)}{\chi(n)g(n)^2}\Big) g(n), \tag{29}$$

where $\mu(n)$ may be interpreted as the return on assets of the firm.

The terms in parentheses correspond to three channels for investor impact, respectively: increasing the firm size $K(n)$ with $g(n)$ fixed, increasing firm size via an increase in $\Delta g(n)$, and increasing $\Delta g(n)$ given fixed $K(n)$. The contribution multiplier $\mathcal{C}(n,n)$ applies to *all* of these channels, at least for the microfounded case of Section 2. The first channel is the only channel for investor impact in my baseline model. The second channel will be stronger for firms where the endogenous increase in their social productivity ($\Delta g(n)$) is high as a percentage of their social productivity. The third channel will be stronger for firms with small costs to improve social productivity ($\chi(n)$), high net productivity ($\mu(n)$) and low social productivity ($g(n)$). However, Berg et al. (2023) discuss how their results may be indicative of most firms having high costs of improvement except in the governance dimension.

5.2 Model with search frictions

In this subsection, I present a simple model with search frictions to illustrate that contribution multipliers arise in this alternative setting. This relates to Carter et al. (2021)'s call for "probabilistic" approaches to additionality.

Consider a firm in my general baseline setting but suppose that this firm can only take investment from one investor (e.g. a private equity buyout). The investor that they meet and receive investment from is randomly drawn at $t = 0$ from among the $\mathcal{I}_n$ investors that have the firm in their opportunity sets. Investor i must pay a fixed cost to include the firm in their opportunity set. In addition to potential financial returns and diversification benefits, how much should the investor be willing to pay for the investor impact of being part of the demand for the firm?

Note that once an investor is selected as part of the search, I assume they invest with a contribution multiplier of 1. Let $\mathcal{I}_{n,0}$ be the set of all candidate investors excluding investor i. Assuming for simplicity that all investors have similar demand elasticities, the expected size of the firm without the investor will be

$$\bar{K} = \frac{1}{|\mathcal{I}_{n,0}|} \sum_{j \in \mathcal{I}_{n,0}} \frac{a_j(n) + g_j(n)}{\tilde{\zeta}_S(n)^{-1} + \tilde{\zeta}_j(n)^{-1}}, \tag{30}$$

and with the investor it will be,

$$\bar{K} = \frac{1}{|\mathcal{I}_{n,0}| + 1} \left(\sum_{j \in \mathcal{I}_{n,0}} \frac{a_j(n) + g_j(n)}{\tilde{\zeta}_S^{-1}(n) + \tilde{\zeta}_j^{-1}(n)} + \frac{a_i(n) + g_i(n)}{\tilde{\zeta}_S(n)^{-1} + \tilde{\zeta}_j(n)}^{-1} \right), \tag{31}$$

for a change of,

$$\Delta \bar{K} \approx \frac{\frac{1}{|\mathcal{I}_{n,0}|+1}}{\tilde{\zeta}_S(n)^{-1} + \tilde{\zeta}_j(n)^{-1}} (\Delta \mu(n)). \tag{32}$$

where $\Delta\mu(n) = a_i(n) + g_i(n) - \frac{1}{|\mathcal{I}_{n,0}|}\sum_{j\in\mathcal{I}_{n,0}} a_j(n) + g_j(n)$. $\Delta\mu(n)$ could arise from investor i being more optimistic than average about the prospects of the firm.

This result suggests the shift may be viewed as being controlled by a multiplier just like my baseline investor contribution multiplier but with $\frac{1}{|\mathcal{I}_{n,0}|+1}$ playing the role of the demand slope in the numerator. Note that, a more complicated version of this model that takes into account the impact on all N firms (e.g. the opportunity cost of investment for the selected investor) could easily be derived using the techniques of Appendix A.

6 Limitations and potential extensions

In this section I discuss the limitations of my model with a view to potential extensions and other possibilities for future research. I have sought to focus my model on three key ideas: non-trivial supply elasticities, contribution multipliers and the price of impact. Packaged altogether these ideas lead to the concept of "impact returns". This means that there are a host of natural extensions that could be important, at least in certain contexts. I describe a selection of key potential extensions below.

My model is a for a single period of an all-equity economy. Betermier et al. (2022) include debt in their model and, in their appendix, show how to extend the model to multiple periods. As such, I don't expect such extensions to be problematic or to cause any high-level changes to my framework. That is not to say such extensions would not be valuable. In particular, I expect that the price of impact could be made endogenous in a multi-period model. It is also likely that the naive optimal solutions to these models will be brittle and "knife edge" without accounting for model uncertainty and robustness considerations such as those discussed in Hansen and Sargent (2011), Bidder and Dew-Becker (2016), and Watson and Holmes (2016), especially uncertainty about impact and social productivities.

More impact-specific considerations that could be examined within the framework of this model include management tastes for the non-market good and nonlinearities in the utility associated with the non-market good (e.g. threshold values). For example, if fixed costs of change are added to the setting of Section 5.1 then this could create demand thresholds necessary for firm management to make changes. It could then be exceptionally high impact for an investor to shift their demand to firms that are near their threshold for change. One purely financial way that thresholds could develop is due to limitations on short selling.

Developing models of blended finance seems like a natural application of my model. This could include applications to advanced market commitments, social impact bonds, impact certificates and similar. The outcomes for an investor with impact preferences could be contrasted between one scenario where they make impact investments and another where they philanthropically fund payments for the desired outcome. The other investors in the market could be assumed to be willing to pay a price for impact or not. More complex extensions could be made to cases where firms are able to issue multiple types of securities—this would allow for applications to impact investments with senior and junior (also known as, "catalytic") tranches.

The study of universal ownership also seems like a natural application of this framework. Indeed, Quigley (2022) presents a framework for the practice of universal ownership that aligns with my model in several ways. They emphasize a focus on leverage points where supply elasticities are particularly high. Though, the potential heterogeneity that I seek to emphasize with my results suggests relative optimism about the potential for some investors to generate asset allocation impact in listed equities. They argue that traditional responsible investment tends to aim to protect individual portfolios from systemic risks, while universal owners' interest is in actually mitigating these risks—just like the investors in my model (i.e. if the non-market good is a reduction in systemic risk). The cost-sensitivity matrix Σ_C of Section 2 is one way to go beyond standard portfolio theory and capture the direct effect of firms on each other. Such features make this a natural framework within which to define a portfolio theory that meets the demands of universal ownership theory.

Gardner and Henry (2023) present a "dual-hurdle framework" for infrastructure investments in poor countries based on considering two alternative uses of capital as an opportunity cost. This aligns with my discussion of how investors with impact preferences will optimally set their price of impact based on a minimum upper bound inferred from their opportunity costs.

While I do not feature an engagement channel in my model, my results may suggests new directions for research in this stream. Do investors target firms that are deemed to have higher enterprise contributions? Do investors strategically target their engagement efforts towards firms that are "under supplied" with engagement, thereby gaining a higher investor (engagement) contribution?

There are also important considerations that I have left out of this paper as they seem likely to only be of interest to the most technical of impact investment researchers and practitioners. First, there is 'impact risk', which would arise if firms' social productivities were uncertain[12]. While variation of a firm's social productivity is a practical reality, I am not aware of data that is suitable to study how investors respond to impact risk (though Avramov et al. (2021a)'s approach of using disagreement could be a proxy for some amount of impact risk). Additionally, as I discuss in Harris (2021a) and Harris (2021b), there are nuanced normative reasons based on moral philosophy to expect that impact risk is a secondary priority. At the very least, how to treat impact risk is a non-trivial topic. Second, it seems reasonable for investors to assign higher value to investments with returns that are negatively correlated with their future price of impact. This will increase the expected level of the non-market good over time (Harris, 2021a; Harris, 2021b). I refer to this concept as 'mission correlation', though it was first introduced to the literature as 'mission hedging' in Roth Tran (2019). Baker et al. (2022b) focus on hedging climate risk outcomes, in a way that seems distinct from, but similar to mission correlation.

7 Conclusion

This paper models capital markets using a supply and demand system in order to study the effects of impact preferences. The effect of investor demand shifts, and the optimal such demand shifts, depends on the aggregate supply and demand elasticities. Firms with higher supply elasticities and lower associated demand elasticities offer better opportunities for investing for impact.

Reexamining empirical results from the literature, I find that the observed impact returns and prices of impact associated with prominent ESG metrics are surprisingly high and they dwarf the values suggested by known social costs and charitable cost-effectiveness estimates. I also illustrate how investor contribution can vary by orders of magnitude depending on the demand and supply elasticities of the firm in question.

The tension between my observed and 'bottom-up' price of impact estimates highlights several directions for further research. First, estimates of the equilibrium system that account

[12] While I interpret risk to mean the volatility of a firm's contribution to the non-market good, I have observed that many impact investors use 'impact risk' to mean the probability that a firm's contribution will be realized. In my model this probability is already factored into $g(n)$.

for the heterogeneity of supply and demand elasticities could dissolve the tension, if investors are systematically tilting their portfolios towards firms with both better ESG scores and higher investor contribution.

Second, in general, heterogeneity in firm supply and demand elasticities seems understudied, with the traditional assumptions in asset pricing being that the former is zero and that the latter is relatively homogeneous across firms. More estimates of how these parameters vary across firms, asset classes and geographies would help guide address policy questions around impact investing.

Third, my general model offers a framework for exploring a wide range of extensions. This includes direct extensions, such as adding 'impact thresholds', as well as developing quantitative models for approaches to impact that were out of scope for this paper, such as shareholder engagement. It also includes further study of complex concepts and themes like enterprise impact, investor impact, blended finance and universal ownership.

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A Investor Impact

A.1 Proof of proposition 1.1

When $\boldsymbol{K}_i$ changes to $\boldsymbol{K}_i' = \boldsymbol{K}_i + \Delta \boldsymbol{K}_i$, market clearing is maintained by a change in prices. Differentiating both sides of the market clearing condition with respect to $\Delta \boldsymbol{K}_i$ yields

$$\boldsymbol{I} + \sum_{i=1}^{I} \frac{\partial \boldsymbol{K}_j}{\partial \boldsymbol{P}} \frac{\partial \boldsymbol{P}}{\partial \Delta \boldsymbol{K}_i} = \frac{\partial \boldsymbol{K}_S}{\partial \boldsymbol{P}} \frac{\partial \boldsymbol{P}}{\partial \Delta \boldsymbol{K}_i}. \tag{33}$$

Rearranging this becomes

$$\frac{\partial \boldsymbol{P}}{\partial \Delta \boldsymbol{K}_i} = (\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1}. \tag{34}$$

This implies that the derivative of $\boldsymbol{K}$ with respect to $\Delta \boldsymbol{K}_i$ is

$$\boldsymbol{\mathcal{C}} \equiv \frac{\partial \boldsymbol{K}}{\partial \boldsymbol{K}_i} = \tilde{\boldsymbol{\zeta}}_S \frac{\partial \boldsymbol{P}}{\partial \boldsymbol{K}_i} = \tilde{\boldsymbol{\zeta}}_S (\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1}. \tag{35}$$

Thus, to first order, the investor impact of a demand shift $\Delta \boldsymbol{K_i}$ is

$$II_i(n) = \boldsymbol{g}' \Delta \boldsymbol{K} = \boldsymbol{g}' \frac{\partial \boldsymbol{K}}{\partial \boldsymbol{K}_i} \Delta \boldsymbol{K_i} = \boldsymbol{g}' \boldsymbol{\mathcal{C}} \Delta \boldsymbol{K_i}. \tag{36}$$

A.2 Inverse matrix decomposition

Suppose we would like to understand the investor impact of a change in demand for firm n, $\Delta K_i(n)$. Without loss of generality, assume $n = 1$ (i.e. is the first element in $\boldsymbol{K}$). Let $\tilde{\boldsymbol{\mathcal{M}}}_D = \tilde{\boldsymbol{\zeta}}_D^{-1}$, $\tilde{\boldsymbol{\mathcal{M}}}_S = \tilde{\boldsymbol{\zeta}}_S^{-1}$, $\tilde{\boldsymbol{\mathcal{M}}}_T = \tilde{\boldsymbol{\mathcal{M}}}_D + \tilde{\boldsymbol{\mathcal{M}}}_S$ and $\boldsymbol{\beta}_{M,n} = (\tilde{\boldsymbol{\mathcal{M}}}_T(-n,-n))^{-1} \tilde{\boldsymbol{\mathcal{M}}}_T(-n,n)$. The inverse of $\tilde{\boldsymbol{\mathcal{M}}}_T = \tilde{\boldsymbol{\mathcal{M}}}_D + \tilde{\boldsymbol{\mathcal{M}}}_S$ may be decomposed using the following analytic blockwise inversion formula:

$$\tilde{\boldsymbol{\mathcal{M}}}_T^{-1} = \begin{pmatrix} \tilde{\boldsymbol{\mathcal{M}}}_T(n,n) & \tilde{\boldsymbol{\mathcal{M}}}_T(-n,n) \\ \tilde{\boldsymbol{\mathcal{M}}}_T(n,-n) & \tilde{\boldsymbol{\mathcal{M}}}_T(-n,-n) \end{pmatrix}^{-1} \tag{37}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & (\tilde{\boldsymbol{\mathcal{M}}}_T(-n,-n))^{-1} \end{pmatrix} + \begin{pmatrix} 1 \\ -\boldsymbol{\beta}_{T,n} \end{pmatrix} (\tilde{\boldsymbol{\mathcal{M}}}_T(n,n) - \tilde{\boldsymbol{\mathcal{M}}}_T(n,-n)\boldsymbol{\beta}_{T,n})^{-1} \begin{pmatrix} 1 \\ -\tilde{\boldsymbol{\beta}}_{T,n} \end{pmatrix}',$$

with

$$\boldsymbol{\beta}_{T,n} = (\tilde{\boldsymbol{\mathcal{M}}}_T(-n,-n))^{-1}\tilde{\boldsymbol{\mathcal{M}}}_T(-n,n), \tag{38}$$

$$\tilde{\boldsymbol{\beta}}_{T,n} = (\tilde{\boldsymbol{\mathcal{M}}}_T(-n,-n))^{-1}\tilde{\boldsymbol{\mathcal{M}}}_T(n,-n)', \tag{39}$$

where subscripts with a single n represent single rows and columns, and subscripts with $(-n)$ refer to all rows or columns excluding n.

A.3 Enterprise impact

Suppose that the aggregate supply and demand curves are $\boldsymbol{P} = \boldsymbol{\mu}_S + \boldsymbol{\mathcal{M}}_S\boldsymbol{K}$ and $\boldsymbol{P} = \boldsymbol{\mu}_D - \boldsymbol{\mathcal{M}}_D\boldsymbol{K}$. Let $\boldsymbol{\mu}_T \equiv \boldsymbol{\mu}_D - \boldsymbol{\mu}_S$. In equilibrium the capital amounts will be $\boldsymbol{K} = \boldsymbol{\mathcal{M}}_T^{-1}\boldsymbol{\mu}_T$, so that

$$K(n) = (\mathcal{M}_T(n,n) - \mathcal{M}_T(n,-n)\boldsymbol{\beta}_{T,n})^{-1} \begin{pmatrix} 1 \\ -\tilde{\boldsymbol{\beta}}_{T,n} \end{pmatrix}' \boldsymbol{\mu}_T \tag{40}$$

Define the counterfactual Enterprise Impact (EI) of a firm's existence as the difference in $G = \boldsymbol{g}'\boldsymbol{K}$ between an economy with and without firm n. Assuming no changes to the demand and supply curves after the removal of the firm, the Enterprise Impact is

$$EI_n = \boldsymbol{g}'\boldsymbol{\mathcal{M}}_T^{-1}\boldsymbol{\mu}_T - \boldsymbol{g}'_{(-n)}\boldsymbol{\mathcal{M}}_T(-n,-n)^{-1}\boldsymbol{\mu}_T(-n) \tag{41}$$

$$= \begin{pmatrix} g(n) \\ \boldsymbol{g}_{(-n)} \end{pmatrix}' \begin{pmatrix} 1 \\ -\boldsymbol{\beta}_{T,n} \end{pmatrix} (\mathcal{M}_T(n,n) - \mathcal{M}_T(n,-n)\boldsymbol{\beta}_{T,n})^{-1} \begin{pmatrix} 1 \\ -\tilde{\boldsymbol{\beta}}_{T,n} \end{pmatrix}' \boldsymbol{\mu}_T \tag{42}$$

$$= g(n)\mathcal{C}^E(n)K(n), \tag{43}$$

with

$$\mathcal{C}^E(n) = \left(1 - \frac{\boldsymbol{g}'_{(-n)}}{g(n)}\boldsymbol{\beta}_{T,n}\right), \tag{44}$$

the 'enterprise contribution multiplier'.

A.4 Investor impact

Applying the above inverse matrix decomposition to $\boldsymbol{\mathcal{C}} = \tilde{\boldsymbol{\mathcal{M}}}_T^{-1}\tilde{\boldsymbol{\mathcal{M}}}_D$ to get the change $\Delta K(n)$ caused by a change $\Delta K_i(n)$ yields

$$\begin{aligned}
\Delta K(n) &= \begin{pmatrix}1\\0\end{pmatrix}' \boldsymbol{\mathcal{C}} \begin{pmatrix}\Delta K_i(n)\\0\end{pmatrix} \\
&= \begin{pmatrix}1\\-\tilde{\boldsymbol{\beta}}_{T,n}\end{pmatrix}' \tilde{\boldsymbol{\mathcal{M}}}_D(\cdot,n)\frac{\Delta K_i(n)}{(\tilde{\boldsymbol{\mathcal{M}}}_T(n,n) - \tilde{\boldsymbol{\mathcal{M}}}_T(n,-n)\boldsymbol{\beta}_{T,n})} \\
&= \mathcal{C}^I(n)\Delta K_i(n)
\end{aligned} \tag{45}$$

with "investor contribution multiplier",

$$\mathcal{C}^I(n) = \mathcal{C}(n,n) = \frac{\tilde{\mathcal{M}}_D(n,n)\left(1 - \frac{\tilde{\boldsymbol{\mathcal{M}}}_D(n,-n)}{\tilde{\mathcal{M}}_D(n,n)}\boldsymbol{\beta}_{T',n}\right)}{\tilde{\mathcal{M}}_T(n,n)\left(1 - \frac{\tilde{\boldsymbol{\mathcal{M}}}_T(n,-n)}{\tilde{\mathcal{M}}_T(n,n)}\boldsymbol{\beta}_{T,n}\right)}. \tag{46}$$

Similarly, for investor impact,

$$\begin{aligned}
II_i &= \boldsymbol{g}'\boldsymbol{\mathcal{C}}\Delta\boldsymbol{K}_i \\
&= \sum_{n=1}^{N} \begin{pmatrix}g_i(n)\\\boldsymbol{g}_i(-n)\end{pmatrix}' \begin{pmatrix}1\\-\boldsymbol{\beta}_{T,n}\end{pmatrix} \begin{pmatrix}1\\-\tilde{\boldsymbol{\beta}}_{T,n}\end{pmatrix}' \tilde{\boldsymbol{\mathcal{M}}}_D(\cdot,n)\frac{\Delta K_i(n)}{(\tilde{\boldsymbol{\mathcal{M}}}_T(n,n) - \tilde{\boldsymbol{\mathcal{M}}}_T(n,-n)\boldsymbol{\beta}_{T,n})} \\
&= \sum_{n=1}^{N} g_i(n)\mathcal{C}_i^E(n)\mathcal{C}^I(n)\Delta K_i(n).
\end{aligned} \tag{47}$$

B General solution

To first order the effect of a change $\Delta \boldsymbol{K}_i$ on $U_i(\boldsymbol{K}_i, \boldsymbol{P}, \boldsymbol{K})$ is

$$\Delta U_i \approx \left(\frac{\partial U_i}{\partial \boldsymbol{K}_i} + \frac{\partial U_i}{\partial \boldsymbol{P}} \frac{\partial \boldsymbol{P}}{\partial \Delta \boldsymbol{K}_i} + \frac{\partial U_i}{\partial \boldsymbol{K}} \frac{\partial \boldsymbol{K}}{\partial \Delta \boldsymbol{K}_i} \right) \Delta \boldsymbol{K}_i \tag{48}$$

$$= \left(\frac{\partial U_i}{\partial \boldsymbol{K}_i} + \frac{\partial U_i}{\partial \boldsymbol{P}} (\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1} + \frac{\partial U_i}{\partial \boldsymbol{K}} \boldsymbol{\mathcal{C}} \right) \Delta \boldsymbol{K}_i \tag{49}$$

The first order condition comes from setting the term in parentheses to zero (and taking the transpose keeps the result in my default column orientation):

$$\left(\frac{\partial U_i}{\partial \boldsymbol{K}_i} + \frac{\partial U_i}{\partial \boldsymbol{P}} (\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1} + \frac{\partial U_i}{\partial \boldsymbol{K}} \boldsymbol{\mathcal{C}} \right)' = 0 \tag{50}$$

For $U_i = U_i^H(\boldsymbol{K_i}) + U_i^G(\boldsymbol{K})$ the partial derivatives are

$$\frac{\partial U_i}{\partial \boldsymbol{K_i}} = \boldsymbol{h}_i', \tag{51}$$

$$\frac{\partial U_i}{\partial \boldsymbol{P}} = 0, \tag{52}$$

$$\frac{\partial U_i}{\partial \boldsymbol{K}} = \gamma_i^g \boldsymbol{g}_i'. \tag{53}$$

Plugging the above partial derivatives into the first order condition, rearranging and accounting for the original demand curves $\boldsymbol{P}_{i,0}$ yields

$$\boldsymbol{P} = \boldsymbol{P}_{i,0}(\boldsymbol{K}_i, \boldsymbol{K}, \boldsymbol{X}) + \frac{1}{1 + r_f} \left(\boldsymbol{h}_i + \gamma_i^g \boldsymbol{\mathcal{C}}' \boldsymbol{g}_i \right). \tag{54}$$

C Microfounded solution

C.1 Investor demand curves

In this subsection all vectors and matrices should be understood to apply only over the investor's opportunity set $\mathcal{N}_i$.

Investor i's financial wealth at $t = 1$ is

$$W_{i,1} = W_{i,0}(1 + r_f) + \boldsymbol{z}'_{CF} \operatorname{diag}(\boldsymbol{P})\boldsymbol{K}_i - (1 + r_f)\boldsymbol{P}'(\boldsymbol{K}_i - \boldsymbol{K}_{i,0}). \tag{55}$$

The part of each investor's utility that is relevant to the solution of the model is,

$$\tilde{U}_i = \boldsymbol{a}'_i\boldsymbol{K}_i - (1 + r_f)\boldsymbol{P}'(\boldsymbol{K}_i - \boldsymbol{K}_{0,i}) - \frac{\gamma_i}{2W_{i,0}}\boldsymbol{K}'_i\boldsymbol{\Sigma}_{CF,i}\boldsymbol{K}_i + \gamma_i^g\boldsymbol{g}'_i\boldsymbol{K}. \tag{56}$$

The relevant partial derivatives of the utility function are:

$$\frac{\partial U_i}{\partial \boldsymbol{K_i}} = \left(\boldsymbol{a}_i - (1 + r_f)\boldsymbol{P} - \frac{\gamma_i}{W_{i,0}}\boldsymbol{\Sigma}_{CF,i}\boldsymbol{K}_i\right)', \tag{57}$$

$$\frac{\partial U_i}{\partial \boldsymbol{P}} = -(1 + r_f)(\boldsymbol{K}_i - \boldsymbol{K}_{0,i})', \tag{58}$$

$$\frac{\partial U_i}{\partial \boldsymbol{K}} = \gamma_i^g\boldsymbol{g}'_i. \tag{59}$$

Here I have ignored that $W_{0,i}$ is endogenous as accounting for this only produces second-order terms (see discussion in Subsection C.4 below).

The first order condition for the demand curves is thus

$$\boldsymbol{a}_i - (1 + r_f)\boldsymbol{P} - \frac{\gamma_i}{W_{i,0}}\boldsymbol{\Sigma}_{CF,i}\boldsymbol{K}_i - (1 + r_f)(\boldsymbol{K}_i - \boldsymbol{K}_{0,i})(\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1} + \gamma_i^g\boldsymbol{\mathcal{C}}'\boldsymbol{g}_i = 0. \tag{60}$$

Rearranging, and noting that for an infinitesimal investor $(\boldsymbol{K}_i - \boldsymbol{K}_{0,i}) \to 0$ while $\boldsymbol{K_i}/W_{0,i}$ does not, this implies the optimal demand curve for the investor is

$$\boldsymbol{P} = \frac{1}{1 + r_f}\left(\boldsymbol{a}_i - \frac{\gamma_i}{W_{i,0}}\boldsymbol{\Sigma}_{CF,i}\boldsymbol{K}_i + \gamma_i^g\boldsymbol{\mathcal{C}}'\boldsymbol{g}_i\right), \tag{61}$$

and that the investor's demand elasticity matrix is

$$\boldsymbol{\zeta}_i = (1 + r_f)\frac{W_{i,0}}{\gamma_i}\boldsymbol{\Sigma}_{CF,i}^{-1}. \tag{62}$$

The aggregate demand elasticity is obtained by summing over the individual demand elasticity matrices, taking care to adapt them to the full opportunity set. Let $\boldsymbol{I}_i$ be as defined in the main text. That is, there is a column for each firm in the economy, a row for each firm n in $\mathcal{N}_i$, and each such row is all zeroes except for a single 1 in the column corresponding to its respective n. Using these indicator matrices the result is given by

$$\boldsymbol{\zeta}_D = \sum_{i=1}^{I} \boldsymbol{I}_i' \boldsymbol{\zeta}_i \boldsymbol{I}_i. \tag{63}$$

C.2 Firm supply curves

The partial derivative of firm n's utility with respect to $\Delta K(n)$ is

$$\frac{\partial U_F(n)}{\partial \Delta K(n)} = -\tilde{\boldsymbol{\zeta}}_D^{-1}(n,n)K(n) + P(n) - (1 + \boldsymbol{\Sigma}_C(n,\cdot)\Delta \boldsymbol{K}). \tag{64}$$

Combining all such derivatives into a vector and setting it to zero implies that

$$\boldsymbol{P} = \mathbf{1} + \operatorname{diag}(\tilde{\boldsymbol{\zeta}}_D^{-1})\boldsymbol{K} + \boldsymbol{\Sigma}_C \Delta \boldsymbol{K} \tag{65}$$

$$= \mathbf{1} + \operatorname{diag}(\tilde{\boldsymbol{\zeta}}_D^{-1})\boldsymbol{K}_0 + (\operatorname{diag}(\tilde{\boldsymbol{\zeta}}_D^{-1}) + \boldsymbol{\Sigma}_C)\Delta \boldsymbol{K}. \tag{66}$$

This implies $\tilde{\boldsymbol{\zeta}}_S = (\operatorname{diag}(\tilde{\boldsymbol{\zeta}}_D^{-1}) + \boldsymbol{\Sigma}_C)^{-1}$. Assuming that capital installation costs dominate the aggregate investor demand elasticity, i.e. $\boldsymbol{\Sigma}_C >>> \operatorname{diag}(\tilde{\boldsymbol{\zeta}}_D^{-1})$, then the this simplifies to $\tilde{\boldsymbol{\zeta}}_S = \boldsymbol{\Sigma}_C^{-1}$ so that the supply curves are

$$\boldsymbol{P} = \mathbf{1} + \operatorname{diag}(\tilde{\boldsymbol{\zeta}}_D^{-1})\boldsymbol{K}_0 + \boldsymbol{\Sigma}_C \Delta \boldsymbol{K} \tag{67}$$

$$= \mathbf{1} - \boldsymbol{\Sigma}_C \boldsymbol{K}_0 + \boldsymbol{\Sigma}_C \boldsymbol{K}. \tag{68}$$

C.3 Equilibrium values

Let the aggregate demand curve be written

$$\boldsymbol{P} = \frac{1}{1+r_f}(\boldsymbol{a} + \gamma^g \boldsymbol{\mathcal{C}}' \boldsymbol{g}) - \tilde{\boldsymbol{\zeta}}_D^{-1} \boldsymbol{K}, \tag{69}$$

where the vectors and matrices of aggregate parameters are constructed by multiplying the investor level parameters by $(\boldsymbol{I}_i)'(\boldsymbol{I}_i \boldsymbol{\zeta}_i (\boldsymbol{I}_i)')^{-1} \boldsymbol{I}_i$, aggregating and reversing the multiplication.

Solving for the equilibrium based on the supply and demand curves, the firm sizes are

$$\boldsymbol{K} = (\tilde{\boldsymbol{\zeta}}_D^{-1} + \tilde{\boldsymbol{\zeta}}_S^{-1})^{-1}\left(\frac{1}{1+r_f}(\boldsymbol{a} + \gamma^g \boldsymbol{\mathcal{C}}'\boldsymbol{g}) - \boldsymbol{1} - \tilde{\boldsymbol{\zeta}}_S^{-1}\boldsymbol{K}_0\right) = (\tilde{\boldsymbol{\zeta}}_D^{-1} + \tilde{\boldsymbol{\zeta}}_S^{-1})^{-1}\boldsymbol{\mu} \tag{70}$$

with $\boldsymbol{\mu} = \frac{1}{1+r_f}(\boldsymbol{a} + \gamma^g \boldsymbol{\mathcal{C}}'\boldsymbol{g}) - \boldsymbol{1} - \tilde{\boldsymbol{\zeta}}_S^{-1}\boldsymbol{K}_0$.

Equilibrium prices are

$$\boldsymbol{P} = (\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1}\left(\frac{\tilde{\boldsymbol{\zeta}}_D}{1+r_f}(\boldsymbol{a} + \gamma^g \boldsymbol{\mathcal{C}}'\boldsymbol{g}) + \tilde{\boldsymbol{\zeta}}_S\boldsymbol{1} + \boldsymbol{K}_0\right), \tag{71}$$

$$\approx (\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1}\left(\frac{\tilde{\boldsymbol{\zeta}}_D}{1+r_f}(\boldsymbol{a} + \gamma^g \boldsymbol{\mathcal{C}}'\boldsymbol{g}) + \boldsymbol{K}_0\right) \tag{72}$$

Assuming the supply elasticities are much smaller than the demand elasticities, and dropping $\boldsymbol{K}_0$ to focus on the endogenous effects results in

$$\boldsymbol{P} \approx \frac{(\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1}\tilde{\boldsymbol{\zeta}}_D}{1+r_f}(\boldsymbol{a} + \gamma^g \boldsymbol{\mathcal{C}}'\boldsymbol{g}) \tag{73}$$

C.4 Effect of endogenous investor wealth

Accounting for $W_{i,0} = \boldsymbol{P}'\boldsymbol{K}_{i,0} + C_i$ the corresponding vector of derivatives with respect to the price shift $\frac{\partial dU_i}{\partial d\boldsymbol{P}}$ to investor i's utility resulting from the shift then includes additional terms $(1 + r_f + \frac{\gamma_i}{2W_{i,0}^2}(\boldsymbol{K}_i'\boldsymbol{\Sigma}_{CF,i}\boldsymbol{K}_i))\boldsymbol{K}_{i,0}$. Assuming the investor was and continues to be approximately diversified across the N firms, this additional term will be small for small $W_{i,0}$ as $\boldsymbol{K}_{i,0} \to 0$. Hence, while the investor's initial share holdings could affect their behavior, this effect should be insignificant in the baseline case with infinitesimal investors.

D Endogenous social productivities

I drop the indexes (n), and (n, n) for matrices, throughout this subsection.

To solve the model in this case I assume that each investor's demand curve may be written as

$$P = P_{i,0} - \frac{\chi}{2}(\Delta g)^2 + \mathcal{M}_i^g g \tag{74}$$

and, thus, that the aggregate demand curve is

$$P = P_0 - \frac{\text{diag}(\chi)}{2}(\Delta g)^2 + \zeta_D^{-1}\mathcal{M}^g g \tag{75}$$

where $\mathcal{M}_i^g$ is a coefficient to be optimized and $\mathcal{M}^g = \sum_{i=1}^{I} \zeta_i \mathcal{M}_i^g$.

First I solve for the firm's policy. I assume each firm seeks to maximize demand by setting $\frac{\partial K_D(n)}{\partial \Delta g(n)} = -\zeta_D(n,n)\chi(n)\Delta g(n) + \mathcal{M}^g(n,n) = 0$ as this maximizes their economic profit. The solution is given by

$$\Delta g(n) = \frac{\mathcal{M}^g}{\zeta_D \chi}. \tag{76}$$

Now I solve for the investor's optimal $\mathcal{M}_i^g$ to first order. The partial derivatives of the key model variables with respect to $\mathcal{M}_i^g$ are as follows:

$$\frac{\partial K_i}{\partial \mathcal{M}_i^g} \approx \zeta_i g, \tag{77}$$

$$\frac{\partial g}{\partial \mathcal{M}_i^g} = \zeta_D^{-1}\chi^{-1}\zeta_i, \tag{78}$$

$$\frac{\partial P}{\partial \mathcal{M}_i^g} = (\zeta_S + \zeta_D)^{-1}\zeta_i(\mathcal{M}^g \chi^{-1} + g), \tag{79}$$

$$\frac{\partial K}{\partial \mathcal{M}_i^g} = \mathcal{C}\zeta_i(\mathcal{M}^g \chi^{-1} + g). \tag{80}$$

The corresponding partial derivatives of the investor's utility are:

$$\frac{\partial U_i}{\partial K_i} = -\mathcal{M}_i^g g, \tag{81}$$
$$\frac{\partial U_i}{\partial g} = \gamma_i^g K, \tag{82}$$
$$\frac{\partial U_i}{\partial P} = K_i, \tag{83}$$
$$\frac{\partial U_i}{\partial K} = \gamma_i^g g. \tag{84}$$

Combining the derivatives together gives the following approximate first-order condition:

$$0 \approx -\mathcal{M}_i^g g^2 + \gamma_i^g (K\chi^{-1}\tilde{\zeta}_D^{-1} + \mathcal{C}g(\mathcal{M}^g\chi^{-1} + g)) \tag{85}$$

with solution

$$\mathcal{M}_i^g \approx \gamma_i^g \left(\mathcal{C}\Big(1 + \frac{\Delta g}{g}\Big) + \frac{\zeta_D^{-1} K}{\chi g^2} \right) \tag{86}$$

Based on Appendix C, ζ_D^{-1} will be approximately $(1+r_f)\frac{\gamma}{W_0}\sigma_{CF}^2$, so it is possible that ζ_D^{-1} is even smaller than K is large. An approximate model of firm size, based on Appendix C, is that $K \approx (\tilde{\zeta}_D^{-1} + \tilde{\zeta}_S^{-1})^{-1}\mu$ where μ is essentially the return on assets of the firm's technology. But $\mathcal{C} = (\tilde{\zeta}_D^{-1} + \tilde{\zeta}_S^{-1})^{-1}\tilde{\zeta}_D^{-1}$. So with this approximation the contribution multiplier applies to every term and may be brought out of the parentheses

$$\mathcal{M}_i^g \approx \gamma_i^g \mathcal{C}\Big(1 + \frac{\Delta g}{g} + \chi^{-1}\frac{\mu}{g^2}\Big). \tag{87}$$