Notes on risk-neutral pricing of SLBs and step-down structures

AUTHOR NAMES HERE

Abstract

Sustainability-Linked Bonds (SLBs) have quickly risen to as a new way to finance transition plans encompassing a full corporate structure rather than just specific asset. This paper analyses SLBs through a risk-neutral present value scenario approach, assuming that the discounted, probability-weighted cashflows in an SLB should equate to those of an equivalent vanilla bond (EVB). We show how, in a risk-free setting, a step-down SLB must by necessity offer a coupon above that of the equivalent vanilla bond, and vice versa for step-ups. The model allows us also to back-out market-implied step-up probabilities and can be used to adjust a structure to accommodate a sought-after lower cost-of-capital for the issuer. When expanding the model to include various risk perceptions we show that the dynamics change such that a step-down SLB may actually price with a coupon lower than a traditional bond, if the sustainability linkages are correlated with improved credit quality/higher repayment probabilities. We illustrate this by calibrating a step-down SLB to rating based spread curves. Our results outline how SLBs can be used by investors to provide conditional lower cost-of-capital, but without breaching fiduciary duty, and thus potentially become an important tool for fixed income markets to drive climate transition.[1].

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[1] This work should be considered a discussion paper and we invite for comments and critique. Contributions from XXXX are gratefully acknowledged. Remaining errors are the sole responsibility of the authors. Not investment advice, full disclaimers available at Ys

1 Introduction

Since its beginnings in 2019, sustainability-linked bonds have largely been issued in the form of bonds paying coupon step-ups in the event of missed sustainability performance targets (SPTs). Only a handful of issuers have come to market with step-down SLBs.[2] This is in stark contrast to the sustainability-linked loan (SLL) market, where the step-down format is well established.

Although viewed positively by the impact community as an incentive for issuers to set more aggressive targets and a solution to intergenerational alignment issues, investors' appetite has so far been relatively subdued. Uncertainties around the return profile of the instrument with potential future yield drop and subsequent painful discussions with end investors as well as the actual achievement of sustainability impact[3] seem to be barriers to adoption. Noting that, certain parties are accepting SLBs on a similar basis to traditional bonds, such as the European Central Bank (ECB) in its corporate bond purchase program.

Despite some of these hurdles, step-downs start to gather some momentum in both the corporate and sovereign SLB space[4] with issuers such as Uruguay or Malaysia, preparing inaugural SLBs issuances with step-down features. The question remains now on how such features should affect the pricing of SLBs. It is our belief that a better understanding of step-downs' impact on SLB risks and returns can only speed up their adoption from the market, where, for example, Hay (2022) brings up some of the questions investors and issuers currently raise. For earlier discussion on pricing of SLBs, Mielnik and Erlandsson (2022) provide an option-pricing approach which disaggregates the difference in spreads between an SLB and traditional bonds into an option premium component and a non-pecuniary greenium. Earlier literature on SLB greenium pricing is sparse, with Köbel and Lambillon (2022) estimating SLB greeniums (not disaggregating the option premium) to a fairly sizable number. Zerbib (2019) conducts a broad study not on SLBs but on the non-pecuniary feature of green bonds as a reference. Harrison (2022) conducts a study on SLB primary market spread differentials with traditional bond curves, again focusing on a total greenium rather than separating out option premiums.

This paper complements our earlier approach where we use an option-pricing to model expected values of SPTs and derive pricing of SLBs. In contrast, this paper adopts a simpler framework in order to enable a straightforward discussion of relative pricing of different structures: we develop a simple binomial framework to analyse arbitrage-free risk-neutral SLB pricing given fixed proba-

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[2] For example, on 10 Nov 2021, Thai Union issued THB 4.5Bn 5y step-down SLB (TH0450A36B06). On 24 Mar 2022, Hap Seng Management issued MYR 75Mln 3y step-down SLB (MYBUG2201039). A dual step-up, step-down structure was issued by the Uruguay sovereign on 20 Oct 2022 (US760942BE11). An updated list of sustainability-linked bonds can be found either through the Bloomberg terminal or through ICMA's database.

[3] See, for example, Erlandsson (2021) around the SLB of Enbridge, a Canadian oil sands company, but also contrasting examples such as in Richardson (2022) looking at utility company Enel, and Erlandsson, Mielnik, Leigh-Bell and Richardson (2022c), that studies a hypothetical structure from meat producer JBS.

[4] There are a few examples of earlier sovereign SLBs but with step-up features, see Mielnik and Erlandsson (2022a) for an analysis of Chile's SLB issuance in early 2022

bilities for SPTs to be achieved. Using this framework, in a risk-free setting, it is trivial to show that step-up SLBs (SLBUs) should have a lower coupon than equivalent vanilla bonds (EVBs) and that step-down SLBs (SLBDs) should have a higher coupon. As in our earlier work, the SLB-spread (the yield differential between the SLB and the EVB) is highly conditional on step size, step maturity besides the probability to step.[5]

Although this statement goes against the general thinking of the financial community that issuers committing to sustainability targets should be 'rewarded' in the form of lower cost-of-capital, investors buying step-down SLBs are effectively taking a risk of lower future returns and should be financially compensated for such risk. Nonetheless, we argue in this paper that if the investor estimates a lower likelihood to default if the company achieves SPT compared to an agnostic company management, the investor might have an incentive to price a step-down SLB closer to a traditional bond.

Moreover, in contrast to the simpler, risk-free case, if default probabilities are significantly different, the SLB spread may indeed invert, e.g. such that SLBDs could have coupons fixed at a tighter spread/yield than EVBs. This can explain a 'greenium' if the investor estimates a lower likelihood to default if the company achieves the SPT compared to an agnostic company management. This effect has important implications for creating SLB structures with step-downs that actually price inside traditional bonds, which seems an important factor in making the concept attractive to issuers.

2 Pricing SLBs using a risk-neutral approach

We start by assuming that the investor (as well as issuer) in the SLB structure is risk-neutral, meaning that she is not bothered by the increased volatility in future cash-flows that is a direct feature of the structure (as the coupon is variable rather than fixed).[6]

Furthermore, we assume a straight SLB structure, where the observation date of the SPT is at the end of the year 4 (τ), and the coupon step (CS – note that we do not assume a step-up or step-down a priori) happens in year 5 ($\tau + 1$). The structure has a 10 year maturity (T).

Most importantly, we assume a fixed and bilaterally known step probability (p), held at 25% to start with. To be clear, p, represents the probability that the start-date coupon/spread C will be adjusted by the step CS.

Furthermore, we assume a fixed discount rate (r) of 2% per annum. Coupons are paid on an annual basis.

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[5] For an earlier paper on pricing step-up bonds, see Lando and Mortensen (2004).

[6] Erlandsson (2021) discusses the implications of return-risk optimisation in the context of green bond pricing. It would be natural to extend the discussion into SLB structures, and we will pursue this in future work.

Table 1: Present value of EVB, 3.5% coupon, 150bp spread.

Year	0	1	2	3	4	5	6	7	8	9	10	Bond price
EVB	0	1.471	1.442	1.413	1.386	1.359	1.332	1.306	1.280	1.255	1.231	113.474

Table 2: Present value of SLB, base case, 3.5% coupon, 150bp coupon spread, -50bp coupon spread step.

Year	0	1	2	3	4	5	6	7	8	9	10	Bond price
Pre-step, 0…t	0	1.471	1.442	1.413	1.386							
No step, t+1…T						1.359	1.332	1.306	1.28	1.255	1.231	113.474
Step, t+1…T						0.906	0.888	0.871	0.853	0.837	0.82	110.886
Weighted value	0	1.471	1.442	1.413	1.386	1.245	1.221	1.197	1.174	1.151	1.128	112.827

Lastly, we assume that the cash-flows represented by C and $C + CS$ are risk-free – we will alter this assumption in a future paper, but for illustrative purposes, the risk-free assumption makes representation more parsimonious in this first iteration.

Now, consider the price of an Equivalent Vanilla Bond (P_{EVB}), it will simply be the sum of discounted coupons across the life of the bond:

$$P_{EVB} = \sum_{t=1}^{T} \frac{C}{(1+r)^t} + \frac{100}{(1+r)^T} \tag{1}$$

If we assume a coupon (C) of 3.5% and a flat discount curve (r) of 2%, we can price the EVB as in Equation (1) to arrive at a price of 113.474, numerically illustrated in Table 1.

We now assume a step size of an SLB in this context to -50bp ($CS = -0.5\%$), i.e. a step-down of 50bp, and reprice this structure based on the discounted cash flows and the fixed probability that the step will happen. The price of such structure paying a fixed coupon C_{SLB} can be written as:

$$P_{SLB} = \sum_{t=1}^{\tau} \frac{C_{SLB}}{(1+r)^t} + (1-p) \cdot \sum_{t=\tau+1}^{T} \frac{C_{SLB}}{(1+r)^t} + p \cdot \sum_{t=\tau+1}^{T} \frac{CS}{(1+r)^t} + \frac{100}{(1+r)^T} \tag{2}$$

The results from repricing this same structure with $p = 25\%$ are available in Table 1. First, the pre-step up $(1 \ldots \tau)$ cash flows are identical in value to the EVB, but also note that clearly the valuations of the cash-flows in the case of a non-step-up is equivalent to the EVB.

The price differential between the EVB and the SLB does instead (and quite intuitively) arise from the step-change leg of the SLB, which in our case is valued a 110.886. Simple arithmetic gives that, given a probability of 25% to get to a value of 110.886 and 75% to get something with value 113.474 gives us an expected value of 112.827 (the bottom row of Table 1). The intuition here is that an SLB with an identical coupon as an EVB clearly is inferior to the EVB and that should (and is) reflected in the discounted price of the same-coupon SLB.

Table 3: Present value of SLB with a higher 165bp coupon spread, -50bp step.

Year	0	1	2	3	4	5	6	7	8	9	10	Bond price
Pre-step, 0. . . t	0	1.618	1.586	1.555	1.524							
No step, t+1. . . T						1.494	1.465	1.436	1.408	1.381	1.354	114.821
Step, t+1. . . T					0	1.042	1.021	1.001	0.982	0.962	0.943	112.234
Weighted value	0	1.618	1.586	1.555	1.524	1.381	1.354	1.328	1.302	1.276	1.251	114.174

Table 4: Present value of SLB, adjusting coupon spread to 157.2bp to set the price of the SLB equal to the price of the EVB.

Year	0	1	2	3	4	5	6	7	8	9	10	Bond price
Pre-step, 0. . . t	0	1.541	1.511	1.481	1.452							
No step, t+1. . . T						1.424	1.396	1.369	1.342	1.315	1.29	114.121
Step, t+1. . . T						0.971	0.952	0.933	0.915	0.897	0.879	111.533
Weighted value	0	1.541	1.511	1.481	1.452	1.311	1.285	1.26	1.235	1.211	1.187	113.474

Let us first test how the price of the SLB changes as we adjust the initial fixed coupon C. In Table 3, we test to set the coupon spread to 165 rather than the EVB 150bp. Naturally P_{SLB} increases, in this case to 114.174, which clearly gives us $P_{SLB} > P_{EVB}$, indicating that the SLB structure with this higher coupon is to be preferred to the EVB.

If we now assert that risk-neutral investors require the value of the SLB and the EVB to be identical (or they would not invest), we set up the following simple relationship:

$$P_{SLB} = P_{EVB} \tag{3}$$

or simply

$$P_{SLB} - P_{EVB} = 0$$

where we note that the final pricing term (the PV of repayment of notional) cancels out. This simple relationship means we can optimize variables such as initial coupon on the SLB to reverse-engineer a price of the SLB that is equivalent to the price of the EVB.

So what is the fair-value coupon of the SLB (C_{SLB}) such that the relationship in 3 is fulfilled, i.e. such that the prices of the SLB and the EVB equate? We show the results of a numerical optimisation for this in Table 4, seeing that if we set the SLB fixed coupon to 157.2bp and hold our probability for step at 25%, the condition is fulfilled and the probability-weighted present value of the cash flows of the SLB is equal to the vanilla bond. In other words, the investor - under these assumptions - would ask for a 7.2bp premium to enter into the SLB step-down structure.

How would a step-up SLB behave in this model? Not surprisingly (and not shown here), the behavior is completely symmetric, and the step-up SLB would, assuming a 50pb step-up, have a fair-value coupon of 142.8bp, i.e. the investor should accept a 7.2bp reduction in yield/spread to get access to the probability of a higher cash-flow.

Staying on the step-up side, we could ask questions such as, "If the issuer is seeking a lowered cost-of-capital of 5bp through issuing an SLB, what should

Table 5: Valuing an SLB holding the coupon fixed to 145bp and adjusting step-size to 34.72bp.

Year	0	1	2	3	4	5	6	7	8	9	10	Bond price
Pre-step, 0...t	0	1.422	1.394	1.366	1.34							
No step, t+1...T						1.313	1.288	1.262	1.238	1.213	1.19	113.025
Step, t+1...T						1.628	1.596	1.565	1.534	1.504	1.474	114.821
Weighted value	0	1.422	1.394	1.366	1.34	1.392	1.365	1.338	1.312	1.286	1.261	113.474

Table 6: Backing out implied probabilities for a given coupon (165bp).

Year	0	1	2	3	4	5	6	7	8	9	10	Bond price
Pre-step, 0...t	0	1.618	1.586	1.555	1.524							
No step, t+1...T						1.494	1.465	1.436	1.408	1.381	1.354	114.821
Step, t+1...T						1.042	1.021	1.001	0.982	0.962	0.943	112.234
Weighted value	0	1.618	1.586	1.555	1.524	1.259	1.234	1.21	1.186	1.163	1.14	113.474

the coupon step-up be?" We illustrate this in Table 5, where we have optimised using a coupon step-up but held the fixed coupon level steady at the sought 145bp.[7] In this case, we can back-out that the coupon step-up should be 34.72bp in order to satisfy the risk-neutral identity in Equation 3.

Yet another way to look at this is to price out the implied step-probability, assuming that all other factors are fairly valued. Consider the bond in Table 3, an SLBD with a 15bp premium versus the EVB, and where we assumed a step probability of 25%. However, if we reprice that bond with a step probability of 52% instead, as in Table 6, we see that the risk-neutral identity is satisfied. In other words, for the observed coupon and step-ups, the market-implied likelihood for step-up is 52%.

Clearly, what this exercise illustrates is a point we previously made in Mielnik and Erlandsson (2022a), that there is a direct optionality value in SLBs that should be separated from any non-pecuniary 'greenium'. Indeed, in order to claim any inference on the size of the greenium in an SLB, one most first extract the actual fair-value of the SLB structure. Let us assume that we observe an SLB such as that in Figure 2, but we know that the fair-value is as what we observe in Figure 4, then we can say that the differential (7.2bp) is an actual greenium – that is how much investor overpaid to get into the SLB structure.

These simple examples also illustrate that, as could be expected, step-up structures will have fixed coupons inside the traditional curve, and step-down structures wider than the curve, in a *risk-free* setting. In a section further below, we relax the risk assumption, which fundamentally changes the results with regards to if, for example, the step-down SLB can price inside the EVB.

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[7] Again, we assume the step-up probability to be known and constant. Relaxing this and deriving implied step probabilities given some law-of-motion for the underlying probabilities is discussed in detail in Mielnik and Erlandsson (2022a). The paper specifically generates implied step-probabilities by assuming that the KPIs/SPTs follow a geometric Brownian motion, such that the Black-Scholes option pricing methodology can be applied.

3 Step probabilities as a driver of the investor-issuer exchange

The above rather simplistic structure provides a good starting point for understanding more of the actual risk exchanges that go on in SLBs. As we showed in the example in Table 1, the probability to step is a parameter in terms of pricing the whole SLB structure.

From a markets perspective, there might be a divergence in opinions between the investor and the issuer in terms of the probability that the SPTs will (not) be met and consequently if the step will happen. If there is 0% probability that it will happen, the SLB initial coupon should converge to the EVB coupon. We illustrate this, using the same parameters as before, in Figure 1. In the top graph, the step-down probability is zero and the fair-value coupon spread becomes 150bp as in the EVB case. In the bottom graph, the SLB prices as an EVB through $0 \dots \tau$ and then as an vanilla bond with a 100bp coupon spread for $\tau+1 \dots T$. This gives a maximum range of $[150, \dots, 178.8]$bp in this structure.

So if the investor and the issuer perceive the likelihood of a step differently, there is a clear spread differential to be expected here. An investor who believes there is a 50% probability of a step-down sees a fair value at 164.4bp coupon spread, but perhaps the issuer sees a 75% probability and thus estimates the fair value at 171.6bp. This gives arise to a 7bp differential to trade between the parties: an SLB offered at 168bp would seem ‘cheap’ to both sides. This goes analogously for the step-up case, illustrated in the right-hand panel of Figure 8.

Of course, the probability of the SPTs being met will be a direct function of the ambition of those targets: an issuer is less likely (has a lower probability) to reach an ambitious target, which should also translate into the pricing of the initial coupon. From a practical standpoint, this raises the question why one would issue an SLB with ‘participation trophy’ targets,[8] as, from a financial standpoint, there remains very little potential to actually vary from the EVB when probabilities for step-downs are close to 100% and 0% for step-ups.

4 SLB pricing with SPT-conditional default rates

We have so far assumed that the cash-flows in both the EVB and SLB cases are risk-free, i.e. there is no risk for non-repayment of principal and coupon payments. We now consider the case of exogenously given default probabilities, to understand the effects if a company adheres to its sustainability targets and that those targets translate into a change of default/credit risk.[9]

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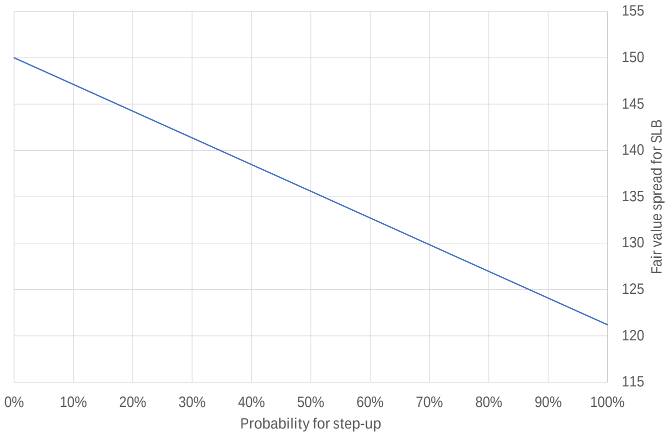
[8] See Erlandsson (2021) as an example. The note highlights an SLB from Canadian oil pipeline company Enbridge, where several of the targets were of a nature deemed to be trivial to be fulfilled.

[9] For avoidance of doubt, we assume that the SLB’s role in the issuer’s capital structure is small, such that the fundamental default probability of the issuer is not more than marginally affected by the conditions of the SLB.

Figure 1: Step-up and step-down probabilities for a given fair-value SLB coupon spread.



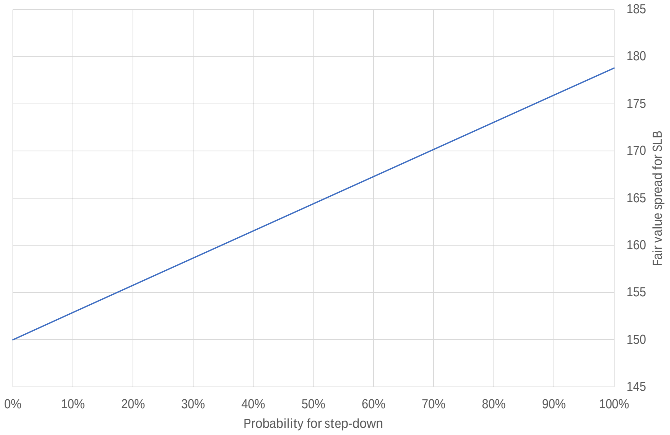


Table 7: Pricing and EVB and SLB with conditional default probabilities: 2% when not stepped, 1.8% when stepped. Step(down) -50bps, coupon 3.5%.

Year	0	1	2	3	4	5	6	7	8	9	No default
EVB											
Scenario prob	0.02	0.02	0.019	0.019	0.018	0.018	0.018	0.017	0.017	0.017	0.817
PV cash flow	40	42.647	45.242	47.786	50.281	52.726	55.124	57.474	59.779	62.038	113.474
Prob weighted	0.8	0.836	0.869	0.9	0.928	0.953	0.977	0.998	1.017	1.034	92.716
										Sum	102.028
SLB											
Scenario probs											
No step	0.02	0.02	0.019	0.019	0.018	0.005	0.004	0.004	0.004	0.004	0.204
Step						0.012	0.012	0.012	0.012	0.011	0.619
PV cash flow											
No step	40	42.647	45.242	47.786	50.281	52.726	55.124	57.474	59.779	62.038	113.474
Step						52.273	54.227	56.142	58.02	59.861	110.886
Prob weighted											
No step	0.8	0.836	0.869	0.9	0.928	0.238	0.244	0.249	0.254	0.259	23.179
Step						0.638	0.65	0.661	0.67	0.679	68.648
										Sum	100.702

Why would we use this perspective? For example, take a coal-based utility company called Enipur. We - identifying ourselves as investors in this case - may infer a certain default probability for Enipur if it stays its coal-dependent course, but another, lower probability if it were to transform into a higher proportion of renewable generation in its power mix. As an investor, if Enipur enters into an SLB structure where the SPTs are connected to the amount/proportion of renewables or some other similar metric, this means that we would expect to observe a correlation between the payment streams in the SLB structure and actual probability of being repaid the nominal investment and bond coupons.[10] A discussion on sustainability and default rates can be found in Aslan et al. (2021), and a broader discussion on the relationship to economic performance in Cek and Eyupoglu (2020).

To analyse these effects, we will assume 40% recovery in case of default. We assume a 2% status quo annual default rate, and 1.8% if the SPTs have been reached by the observation date τ. To align more with market standard – where SPTs have traditionally been set such that is seems highly likely that the SPTs will be met - we now also assume a 75% probability that the coupon will step down. This implies that the issuer has a fairly high probability for a path where default risk is lower in the back five years (1.8% per annum with 75% probability).

We provide the scenario probabilities, the present-value of the cash-flows and probability weighted PVs in Table 7, with an analytical expression for the pricing provided in the Appendix. The top table gives values for holding the coupon the same for the SLBD and the EVB; the bottom table equates price by changing the SLBD coupon.

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[10] A natural question arises why the issuer would not simply make a commitment to sustainability targets to achieve a better risk-profile (and lower cost of funding) without having to do an SLB structure? In case the issuer is able to make (from the investor perspective) a fully certain commitment and provide certainty to execute on that, then the SLB structure becomes unnecessary. However, if there are uncertainties either on commitment or execution - which would seem to be the normal case - then the SLB provides a financial hedge for the investor if commitments (and potential credit improvements) are not fulfilled.

Table 8: Pricing the SLB conditional default rates with a 3.66% coupon to make it risk-neutral to the EVB. Step(down) -50bps, coupon 3.5% on the EVB.

	Year of default										
	0	1	2	3	4	5	6	7	8	9	No default
EVB											
Scenario prob	0.02	0.02	0.019	0.019	0.018	0.018	0.018	0.017	0.017	0.017	0.817
PV cash flow	40	42.647	45.242	47.786	50.281	52.726	55.124	57.474	59.779	62.038	113.474
Prob weighted	0.8	0.836	0.869	0.9	0.928	0.953	0.977	0.998	1.017	1.034	92.716
										Sum	102.028
SLB											
Scenario probs											
No step	0.02	0.02	0.019	0.019	0.018	0.005	0.004	0.004	0.004	0.004	0.204
Step						0.012	0.012	0.012	0.012	0.011	0.619
PV cash flow											
No step	40	42.808	45.56	48.259	50.905	53.498	56.041	58.534	60.979	63.375	114.945
Step						53.045	55.144	57.202	59.22	61.198	112.358
Prob weighted											
No step	0.8	0.839	0.875	0.908	0.939	0.242	0.248	0.254	0.259	0.264	23.48
Step						0.647	0.661	0.673	0.684	0.694	69.559
										Sum	102.028

Table 9: Pricing the SLBD with changed relative default probabilities; 1% in the step-down scenario, 2% otherwise.

	Year of default										
	0	1	2	3	4	5	6	7	8	9	No default
EVB											
Scenario prob	0.02	0.02	0.019	0.019	0.018	0.018	0.018	0.017	0.017	0.017	0.817
PV cash flow	40	42.647	45.242	47.786	50.281	52.726	55.124	57.474	59.779	62.038	113.474
Prob weighted	0.8	0.836	0.869	0.9	0.928	0.953	0.977	0.998	1.017	1.034	92.716
										Sum	102.028
SLB											
Scenario probs											
No step	0.02	0.02	0.019	0.019	0.018	0.005	0.004	0.004	0.004	0.004	0.204
Step						0.007	0.007	0.007	0.007	0.007	0.645
PV cash flow											
No step	40	42.647	45.242	47.786	50.281	52.726	55.124	57.474	59.779	62.038	113.474
Step						52.273	54.227	56.142	58.02	59.861	110.886
Prob weighted											
No step	0.8	0.836	0.869	0.9	0.928	0.238	0.244	0.249	0.254	0.259	23.179
Step						.354	0.364	0.373	0.382	0.39	71.49
										Sum	102.109

As we can see in the base case, the SLB prices significantly below than the EVB and, as the bottom panel illustrates, the SLBD coupon would need to be hiked to 3.66% to compensate the investor for the fairly probable lower coupon cash-flow.

However, this looks quite different if we assume a significantly lower step-down default probability, say 1%, as in Table 9. First, in the top panel, we see that the cash flow valuation of the SLBD is *higher* than for the EVB, 102.109 (SLB) versus 102.028 (EVB). Re-running the risk-neutral valuation (not shown here), would mean a coupon of 3.49%, i.e. 1bp inside the EVB, in this case.

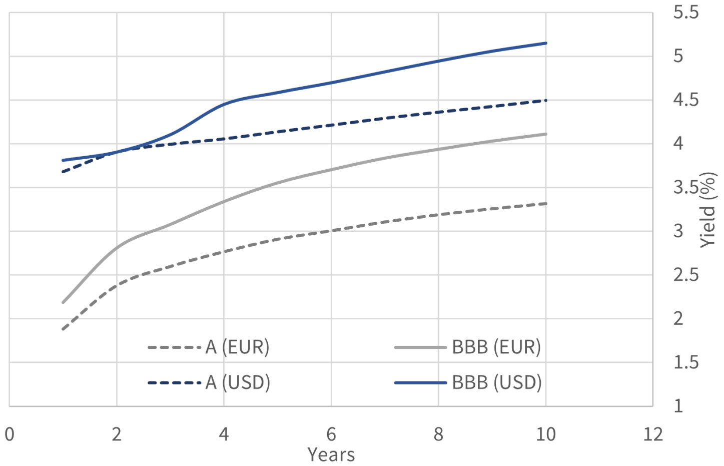
The intuition is straightforward: due to a higher likelihood to be repaid, the investor will still choose the lower coupon stream/cash flow through the SLB. The implications of this are that an issuer that has a convincing enough case in terms of likelihood of reducing default risk in a step-down scenario, can achieve a lower inception coupon on the SLB than the equivalent vanilla bond, i.e. a lower cost-of-capital.

5 Pricing SLBs with SPT-dependent risky discount factors

Although the market-implied default rate approach can be useful to derive market estimates of relative default probabilities, it is harder to use in forward-looking cases, as when looking to price a forthcoming SLB issuance. Our first approach was completely discretionary in terms of assuming a 2% vs 1.8% (a 10% reduction) coupon in case of no-step/step (down); a more calibrated approach would be to express the SLB step effect as a shift in credit ratings. Credit ratings have the advantage of being more stable than spread-implied rates as well as long historical data-sets on realised default rates.

To see how to approach this, assume that the issuer is rated at BBB (flat) with a historical average 1 year default rate for that rating category at 0.16%.[11] Assuming that the issuer is similar to other BBB credit, we can the use the spread curve for bonds as a discount curve, rather than assume anything around default probabilities. One can the make a second assumption, e.g. that if the issuer achieves its SPTs, the credit rating will improve to single-A. For single-As, the historical average 1yr default rate is a mere 0.05%, and only a third of that of BBBs.

Figure 2: Yield curves across rating categories A and BBB.



Now assume that the issuer will receive a full rating letter upgrade in case it reaches the SPTs, i.e. obtains a A rating. We can then replace the discounting curve in case of the SPT with the risky discount factors with the A curve rather than the BBB. We do this in Table 10, using the USD risky curves as seen in Figure 2.

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[11] We take this number from S&P's Default, Transition and Recovery study 2020 here.

Table 10: Valuing the SLBD and EVB using risky discount factor curves. EVB/SLBD coupon 4.5%.

	Year of default										
	0	1	2	3	4	5	6	7	8	9	No default
EVB											
PV cash flow	40	42.867	45.591	48.137	50.457	52.631	54.672	56.583	58.368	60.037	100.051
										Sum	100.051
SLB											
PV cash flow											
No step	40	42.867	45.591	48.137	50.457	52.631	54.672	56.583	58.368	60.037	100.051
Step						52.378	54.197	55.918	57.547	59.089	100.242
										E(Sum)	100.194

Table 11: Pricing risk-neutral equivalent EVB and SLBs using risky discount curves. EVB coupon 4.5%, SLBD coupon 4.482%.

	Year of default										
	0	1	2	3	4	5	6	7	8	9	10
EVB											
PV cash flow	40	42.867	45.591	48.137	50.457	52.631	54.672	56.583	58.368	60.037	100.051
										Sum	100.051
SLB											
PV cash flow											
No step	40	42.85	45.558	48.088	50.392	52.552	54.579	56.476	58.249	59.907	99.908
Step						52.299	54.104	55.811	57.427	58.957	100.098
										E(Sum)	100.051

The result suggests that the SLBD struck at the same coupon as the EVB is more valuable (it prices at 100.194 compared to 100.051). Following the earlier logic, the SLBD offers more value as the higher repayment likelihood more than compensates for a coupon step-down if SPTs are reached. Redoing the same exercise as before, optimising the coupon on the SLB such that the price of the EVB and the SLB are equal, we obtain a coupon of 4.482%, see Table 11.

Of course, this approach does also have a certain degree of arbitrary selection to it ('Why a one notch upgrade?') but seems better to reflect market perception of risk.[12] It also illustrates that the optionality value in the SLB will be time-varying: as risky discount factors vary, so will also the option premium in the SLB. In our case, as illustrated in Figure 2, the EUR curve is quite different from the USD curve. Indeed, when repricing and finding the risk-neutral SLB coupon using the EUR curve, we arrive at an SLB coupon of 4.398%, which is more than 10bps tighter than the equivalent EVB coupon.

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[12] For a discussion on the shape of credit curves of the credit cycle, refer to Rennison et al. (2008).

6 Conclusion

This paper has provided a parsimonious approach to look at SLB pricing under the assumption of a risk-neutral investor. We illustrate how using this approach, one can optimise in different ways to calibrate an SLB structure to sought after standards. This is important in the context of some 'market standards' that have appeared, for example setting coupon steps at 25bps. Our model illustrates the direct relationship between a low coupon step and a (too) high probability to achieve the SPT and the actual cost-of-capital reduction that the issuer can expect to achieve.

In the context of SLB with step-downs, which could be argued are more incentivizing for issuers, we show that in a non-default risk scenario, such structures must price with coupons wider than for the equivalent vanilla bond. However, if we assume that default/repayment probabilities are conditional on the reaching the SPTs, we reach some results where the SLB step-downs structures can indeed price with baseline coupons at tighter levels than the EVB. Of course, this relies on a credible case being provided by the issuer that their repayment capacity will be fundamentally improved by transitioning along the sustainability pathway.

A natural question is how this approach relates to the Black-Scholes approach we proposed in a previous paper. It should be noted that that approach applies a structure to the evolution of the underlying SPTs, and based from those stochastics, we price out the SLB premium, i.e. we directly attempt to find inference on the probabilities of the SPTs being fulfilled. The approach of this paper is to take the step probabilities as given; we will endeavour in future work to combine these approaches.

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8 Appendix

We define X_t as the present value of a standard bond's cash flows in case of default at time t:

$$X_t = \frac{RR}{(1+r)^t} + \sum_{i=0}^{t} \frac{C}{(1+r)^i} - C = \frac{RR}{(1+r)^t} + C \cdot \left[\left(\sum_{i=0}^{t} \frac{1}{(1+r)^i} \right) - 1 \right] \tag{4}$$

Moreover, if we call $a = \frac{1}{1+r}$, we can define the function $f(j)$ as:

$$f(j) = \sum_{i=0}^{j} \frac{1}{(1+r)^i} = \sum_{i=0}^{j} a^i = \frac{1 - a^{j+1}}{1 - a} \tag{5}$$

and write

$$X_t = \frac{RR}{(1+r)^t} + C \cdot [f(t) - 1] \tag{6}$$

We now consider an SLB paying a step-up/down CS after meeting its target at observation date τ. The present value in case of default at time $t > \tau$ can be written as a function of X_t:

$$X_t^{SLB} = \frac{RR}{(1+r)^t} + \sum_{i=0}^{t} \frac{C}{(1+r)^i} + \sum_{i=\tau}^{t} \frac{CS}{(1+r)^i} - C = X_t + CS \cdot [f(t) - f(\tau - 1)] \tag{7}$$

Below we evaluate the price of the SLB paying a step-up/down in case it meets its SPTs at observation τ with probability p^{SLB}.

Considering the bond's default probability is p in case the SPT is not met and p' in case the SPT is met, the SLB price can be expressed as:

$$\begin{gathered} E\left[X^{SLB}\right] = \sum_{i=0}^{t} X_i \cdot p \cdot (1-p)^i + \\ P^{SLB} \cdot \left[\sum_{i=\tau}^{t} X_i^{SLB} \cdot p^t \cdot (1-p^t)^i - \sum_{i=\tau}^{t} X_i \cdot p \cdot (1-p)^i \right] \end{gathered} \tag{8}$$

Considering X_t^{SLB} is a function of X_t, we can write:

$$\begin{gathered} E\left[X^{SLB}\right] = \sum_{i=0}^{t} X_i \cdot p \cdot (1-p)^i + \\ P^{SLB} \cdot \left[\sum_{i=\tau}^{t} X_i \cdot p^* \cdot (1-p)^i - p \cdot (1-p)^i + CS \cdot \sum_{i=\tau}^{t} [f(i) - f(\tau - 1)] \cdot p^* \cdot (1-p^*)^i \right] \end{gathered} \tag{9}$$