Systemic risk effects of climate transition on financial stability

Abstract

We assess how climate transition risk, through its effects on asset prices, could impact financial stability. Using copula functions, we characterize the conditional distribution of financial firm returns under different climate-related market scenarios. We account for average and tail effects of climate transition scenarios to the value of financial firms using three systemic risk metrics: climate transition expected returns, climate transition value-at-risk, and climate transition expected shortfalls. Empirical evidence indicates that European banks experience the highest systemic impacts from a disorderly transition, and that the cost of rescuing more risk-exposed financial firms from climate transition losses is relatively manageable.

J.E.L. Classification: C32, C58, G01, G20, G28

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1. Introduction

Transitioning towards a low-carbon economy entails risks that may impair the performance of firms, with potential ramifications for financial stability. Central banks have warned of the potentially destabilizing effects of climate change risks¹ on financial stability (e.g., the Bank of England, 2017; De Nederlandsche Bank, 2017; ESRB, 2016),² and policymakers have underscored the potential of climate transition as a source of systemic risk.³ Therefore, assessing the impact of climate transition risks on financial firms and on the stability of the financial system is currently a high priority on the agenda of central banks, regulators, and investors (Carney, 2015; European Systemic Risk Board, 2016, Campiglio et al., 2018).

In this paper, we develop an empirical setup to assess the impact of climate transition risk on financial stability. We characterize the distribution of financial firm returns under three different climate transition scenarios: hothouse world, disorderly transition, and orderly transition. Those scenarios account for the potential asset re-pricing effects of climate transition (Carney, 2015). As described by their quantiles, in the hothouse world scenario, the value of highly vulnerable (brown) firms to transition experience upward movements while the value of (green) firms with low vulnerability to transition experience downward movements; in the disorderly transition scenario, green and brown firms experience upward and downward movements, respectively; and in an orderly transition scenario, green, neutral, and brown firm values remain in and around their median values. These movements are coherent with changes in expectations,⁴ and consequently changes in future expected cash-flow from those firms, if the climate transition materializes faster than expected (disorderly transition), slower than expected (hot house world) or its timing is aligned with the climate transition roadmap (orderly transition).

¹ Climate change conveys two main type of risks: (a) physical risk, associated with the impact of extreme weather events such as droughts, floods, hurricanes, etc, and (b) transition risks related to the impact of changes in regulations, business models, technologies, and consumer preferences aimed at being consistent with a low-carbon economy. This research focuses on the effects of transition risks on financial stability.

 $^{^2}$ The concerns of central banks regarding climate-related risk for financial system stability boosted the development of the Network for Greening the Financial System, an initiative of central banks and financial regulators (including the Bank of England and De Nederlandsche Bank). The aim of the NFGS is to foster environment and climate risk management in the financial sector and mobilize mainstream finance to support the transition toward a sustainable economy.

³ See, e.g., <u>https://www.ecb.europa.eu/press/blog/date/2021/html/ecb.blog210318~3bbc68ffc5.en.html</u>.

⁴ Change in expectations could be driven by technological changes, policy changes or changes in the preferences of investors and consumers.

We quantify the impact of each climate transition scenario on financial firm returns in terms of three metrics: the average return of the conditional distribution, a left quantile of the conditional distribution, and the average return below that conditional quantile, labelled climate transition expected returns (CTER), climate transition value-at-risk (CTVaR), and climate transition expected shortfalls (CTES), respectively. The three metrics are computed from the conditional distribution of individual financial firm asset returns, which captures dependence of financial firm returns with green, brown, and neutral asset returns under different climate transition scenarios.

We apply our methodology to European financial firms, including banks, insurance companies, financial services companies, and real estate firms over the period 2013-2020. Our main findings suggest that the systemic impact of climate transition scenarios differs widely across financial institutions. Banks experience more systemic impacts in a disorderly transition than in a hothouse world scenario, while the opposite occurs for the other firm types, but especially for real estate firms. We also find that the systemic impact of the different climate transition scenarios broadly diverges within the financial firm group, yielding potential winners and losers. Southern European financial firms are more exposed to a disorderly transition scenario. These results may be explained by domestic markets in which carbon-intensity firms and energy-efficient firms represent an important share of the market.⁵

We also assess the implications of climate-related systemic risk for financial firms in terms of capital shortfalls (Acharya et al., 2017, Brownless and Engle, 2017). For banks, we document that capital shortfalls are negligible in the orderly transition scenario and sizeable (about 40 billion euros) in the disorderly transition scenario, but concentrated in a small number of entities and so absorbable within the banking sector.⁶ For the remaining financial firms, we find that insurance firms experience small capital shortfalls in any climate transition risk scenario, whereas financial services and real estate firms

⁵ For example, by the end of 2021, 22.31% of the market capitalization in the main Spanish stock index (IBEX35) belongs to NACE sectors with a high exposure to transition risk, whereas only 10% of the market capitalization in the main Finnish stock index (OTM Helsinki) is exposed to this type of climate risk (Alessi and Battiston, 2022). ⁶ This would be in the case of allowing netting within financial firms, i.e., the capital needs of one firm is compensated for by the capital buffer of the remaining firms in the sector.

experience modest capital losses in a hothouse world scenario, but negligible capital losses in the remaining scenarios.

Our study contributes to the literature that addresses the impact of climate-related risks on financial systems. Battiston et al. (2017), for their network-based climate stress-test of climate risk impact in green and brown scenarios, report that European bank exposure to the fossil-fuel sector is small (3%-12%), but is significant and heterogeneous to climate-policy sectors (40%-54%); they also report that the systemic impact of climate risk is expected to be moderate in an orderly transition scenario. Also for Europe, Weyzig et al. (2014) find that the fossil-fuel company revaluation risk for financial stability is limited. Using a calibrated ecological macroeconomic model, Dafermos et al. (2018) argue that climate change is likely to damage the liquidity of firms and negatively affect credit expansion and financial stability, suggesting that those negative climate-induced effects could be reduced by green quantitative easing. Stolbova et al. (2018) report how shocks from the introduction of climate policies generate feedback effects between the real economy and the financial sector that reinforce mispricing and risk transmission. In a recent study of bank exposure to a portfolio of stranded assets, Jung et al. (2023) report a climate stress-testing procedure to measure the climate risk impact on the capital of large global banks, documenting substantial capital shortfalls for most of the studied banks. We add to this literature by introducing a new empirical framework to assess the impact of climate transition risks under different climate-related market scenarios and the implications in terms of tail risk measures and capital shortfalls using realistic (non-Gaussian) modelling assumptions. Our systemic risk measures – which can be readily computed using publicly available market data on individual financial firms and on market assets - can thus reflect changing market conditions, such as induced by the COVID-19 pandemic, and so facilitate timely identification of systemic climate-related risks from a financial stability perspective.

The remainder of the paper is laid out as follows. Section 2 develops our methodological approach, encompassing a definition of climate transition risk metrics, a description of climate transition scenarios, and our empirical modelling approach to quantifying the financial impact of climate transition risks. Section 3 describes data for European financial firms. Section 4 discusses empirical results for the systemic risk impact of the different climate transition scenarios for the European financial system,

while Section 5 provides information on the transition risk implications for capital shortfalls. Finally, Section 7 concludes.

2. Methods

In this section, we first define the CTER, CTVaR and CTES metrics that assess the climate transition risk impact on financial firms. We then outline the climate transition scenarios and the dependence modelling approach to describing and estimating the climate transition risk impact on financial stability under those different scenarios.

2.1 Climate transition risk metrics

Following the systemic risk literature (Acharya et al., 2017; Browless and Engle, 2017; Adrian and Brunnermeier, 2016),⁷ we identify potential vulnerabilities of financial firm returns to different climate transition scenarios using the CTER, CTVaR, and CTES metrics, defined as follows.

DEFINITION 1. CTER is the expected return of financial firm i in the event of a climate transition scenario CT^s :

$$CTER_i = E(r_i | CT^s) = \int_{-\infty}^{\infty} r_i f(r_i | CT^s) dr_i,$$

where r_i denotes the market returns of financial firm *i*, and $f(r_i | CT^s)$ is the density of the returns of financial institution *i* conditional on CT^s .

DEFINITION 2. CTVaR is the maximum possible loss of a financial institution *i* conditional on a climate transition scenario CT^s for a confidence level of $1 - \gamma$:

$$CTVaR_i^{\gamma} = F_{i|CT^s}^{-1}(\gamma),$$

where $F_{i|CT^{s}}^{-1}(\cdot)$ is the inverse cumulative probability distribution of r_{i} conditional on CT^{s} , i.e., $CTVaR_{i}^{\gamma}$ is the γ % quantile of the conditional distribution of returns: $P(r_{i} \leq CTVaR_{i}^{\gamma}|CT^{s}) = \gamma$.

DEFINITION 3. CTES is the expected return of the financial firm *i* when firm returns fall below the $CTVaR_i^{\gamma}$ in the event of a climate transition scenario CT^s :

⁷ For a survey of this literature, see Benoit et al. (2017).

$$CTES_i^{\gamma} = E(r_i | CT^s, r_i \leq CTVaR_i^{\gamma}) = \frac{1}{\gamma} \int_{-\infty}^{CTVaR_i^{\gamma}} r_i f(r_i | CT^s) dr_i.$$

That is, $CTES_i^{\gamma}$ is the expected return value for those returns located in the γ -tail of the conditional distribution of returns.

Figure 1 illustrates how the unconditional distribution of firm i (blue line) shifts in response to the potential impact of a climate transition scenario (red line), along with the above-defined climate transition risk metrics from the conditional firm returns density.

[INSERT FIGURE 1 HERE]

2.2 Climate transition scenarios

We consider three market scenarios, aligned with the narrative of main climate transition scenarios suggested by regulators and supervisory authorities, characterized in terms of their potential asset re-pricing effects: hothouse world (or no transition), disorderly transition to a green economy, and orderly transition to a green economy.⁸

Let r_g , r_n , and r_b denote the market returns of green, neutral, and brown firms, reflecting low, medium, and large vulnerability to climate transition risk, respectively. Hence, the narrative and repricing effects from each transition scenario are as follows.

In the hothouse world scenario, current policies are preserved, emissions grow, and temperatures increase by more than 3°C in a 50-year period. Policy actions to favour transition are implemented slowly and tardily, and investors adjust their expectations accordingly.⁹ In this scenario brown firms have more time to offload stranded assets without suffering a large price impact and their asset prices

⁸ Regulatory and supervisory authorities might consider more than three scenarios for the transition towards a lowcarbon economy, although in terms of market-revaluation we could find a common framework and similar narratives across market authorities for the classification of their scenarios. For instance, the hot house world scenario would entail similar tail market movements that we could expect in the Nationally Determined Contributions and Current Policies scenarios for the NGFS (NGFS, 2021) or the Current Policies Scenario for the Australian Prudential Regulation Authority (Australian Prudential Regulation Authority, 2021) European Central Bank (European Central Bank, 2022), Federal Reserve (Board of Governors of the Federal Reserve System, 2023), Bank of England (Bank of England, 2021) and Bank of Canada (Bank of Canada and Office of the Superintendent of Financial Institutions, 2022). For a comparison between different climate transition scenarios across supervisory authorities, see Table 1 from Acharya et al. (2023).

⁹ Additional changes related with technical innovation and consumer preferences would happen under this scenario, e.g. insufficient climate technological innovation and no relevant consumer preferences for environmentally friendly products in the hot house world. The description of the policy actions is done for illustrative purposes, to depict the umbrella of scenario narratives that could lead to the market outcomes employed to build our climate transition scenarios.

increase, while green asset prices decline as green firms lose the opportunity to boost their business. Thus, the relative price impact of a hothouse world scenario can be described in terms of upward and downward movements in brown and green asset market returns, as described by their quantiles: $r_g \leq q_g^{\alpha}$ and $r_b \geq q_b^{\beta}$, where the α - and β -quantiles of green and brown asset returns are given by $P(r_g \leq q_g^{\alpha}) = \alpha$ and $P(r_b \leq q_b^{\beta}) = 1 - \beta$, respectively. Arguably, the returns of neutral assets experience no particular impact as they are barely affected by the transition risk to a low-carbon economy.

In the disorderly transition scenario, an active stance is adopted through climate policies aimed at mitigating emissions and limiting global warming below 2°C above pre-industrial levels by 2070; however, the implementation of measures is delayed in time, so those policies are introduced abruptly to achieve the emissions goal, resulting in higher transition risks. Abrupt policy constraints on the use of carbon intensive energy may cause frictions in the transition, generating operational difficulties for firms that are more exposed to risk, ultimately affecting the value of their assets (e.g., assets may become stranded). In contrast, firms with lower exposure to transition risk face a privileged position in the market in the short-term. As a result, market expectations regarding green asset prices curve upwards, while the opposite happens with brown asset prices. This impact can be described in terms of upward and downward movements of green and brown asset market returns, characterized by their quantiles: $r_g \ge q_g^{\beta}$ and $r_b \le q_b^{\alpha}$, where the α - and β -quantiles of green and brown asset returns are given by $P(r_b \le q_b^{\alpha}) = \alpha$ and $P(r_g \le q_g^{\beta}) = 1 - \beta$, respectively. As with the hothouse world scenario, the impact of a disorderly transition on neutral asset returns is negligible, as those returns are barely affected by transition risks.

Finally, in the orderly transition scenario, climate policies aimed at keeping global warming below 2°C in the next 50 years are implemented smoothly, allowing firms to progressively adapt to the new business setting. In this context, the transition risk is moderate; since all firms will be able to gradually adapt to the new setup, their values are not expected to experience abrupt changes. Investors would therefore expect asset returns to move around their median values (i.e., with no abrupt price changes),

described as: $q_b^L \le r_b \le q_b^U$, $q_n^L \le r_n \le q_n^U$ and $q_g^L \le r_g \le q_g^U$, where q_j^L and q_j^U are the lower and upper quantiles around the median for the asset j = g, n, b.

2.3 Modelling the financial impacts of climate transition risks

The CTER, CTVaR, and CTES metrics under the three climate transition scenarios are presented in Table 1. Empirical estimation of those metrics requires knowledge of the joint density of the returns of financial firm i and the climate transition scenario, and the probability of that climate transition scenario unfolding (that is, of the conditional density for each financial institution).

[INSERT TABLE 1 HERE]

We characterize the probability distribution of returns using copula functions.¹⁰ Copulas allow marginal and dependence features to be connected, in such a way that the probability distribution of two market returns can be expressed in terms of a bivariate copula function *C* as $F(r_j, r_h) = C_{jh}(F_j(r_j), F_h(r_h))$, where *C* is a cumulative distribution copula with uniform marginal variables given by $F_j(r_j) = u_j$, $F_h(r_h) = u_h$, and where $F_j(r_j)$ and $F_h(r_h)$ denote the marginal distribution function of the *j* and *h* stock returns that stem from the corresponding densities, $f_j(r_j)$ and $f_h(r_h)$. Likewise, the conditional marginal distribution can be obtained from the conditional copula function as $F_{j|h}(r_j|r_h) = C_{j|h}(u_j|u_h) = \frac{\partial C_{jh}(u_j,u_h)}{\partial u_h}$. Copulas can also be extended to the multivariate case, i.e., the probability distribution for the trivariate case can be written in terms of a copula function as $F(r_j, r_h, r_k) = C_{jhk}(F_j(r_j), F_h(r_h), F_k(r_k))$, while the conditional marginal distribution for two variables or one variable is obtained from the conditional copula as $F_{jh|k}(r_j, r_h|r_k) = C_{jh|k}(u_j|u_k, u_h|u_k)$ and $F_{j|hk}(r_j|r_h, r_k) = C_{j|hk}(u_j|(u_h, u_k))$, with $u_j|u_k = C_{j|k}(u_j|u_k)$ and $u_j|(u_h, u_k) = C_{j|kk}(u_j|u_k)(u_k|u_h)$. By separating marginal and joint dependence features, copulas flexibly model multivariate distributions, reporting information on conditional dependence, joint tail dependence, and

¹⁰ For a detailed analysis of copulas, see Joe (1997) and Nelsen (2006).

nonlinearities to accurately assess the systemic impact of tail events such as extreme climate transition scenarios.¹¹

Using copulas, we can express the probability of each climate transition scenario and the joint density between that scenario and the returns of financial firm *i* as follows.

Result 1. The probability of a disorderly transition scenario is given by:

$$P\left(r_g \ge q_g^{\beta}, r_b \le q_b^{\alpha}; r_n\right) = \int_0^1 C\left(u_b \le \alpha, u_g \ge 1 - \beta |u_n\right) du_n,$$
(1)

where $C(u_b \le \alpha, u_g \ge 1 - \beta | u_n) = C_{b|n}(\alpha | u_n) - C_{bg|n}(C_{b|n}(\alpha | u_n), C_{g|n}(1 - \beta | u_n)).$ The

probability for the hothouse world scenario is computed by swapping around the green and brown subscripts. The probability for an orderly transition scenario is given by:

$$P(q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L, q_n^U \ge r_n \ge q_n^L)$$

= $\int_{0.5 - \frac{\delta}{2}}^{0.5 + \frac{\delta}{2}} C\left(0.5 - \frac{\alpha}{2} \le u_b \le 0.5 + \frac{\alpha}{2}, 0.5 - \frac{\beta}{2} \le u_g \le 0.5 + \frac{\beta}{2} |u_n\right) du_n,$ (2)

where:

$$C\left(0.5 - \frac{\alpha}{2} \le u_b \le 0.5 + \frac{\alpha}{2}, 0.5 - \frac{\beta}{2} \le u_g \le 0.5 + \frac{\beta}{2} |u_n\right) = C_{bg|n}\left(C_{b|n}(a|u_n), C_{g|n}(b|u_n)\right) + C_{bg|n}\left(C_{b|n}(a|u_n), C_{g|n}(e|u_n)\right) - C_{bg|n}\left(C_{b|n}(a|u_n), C_{g|n}(e|u_n)\right) - C_{bg|n}\left(C_{b|n}(a|u_n), C_{g|n}(e|u_n)\right) + C_{bg|n}\left(C_{b|n}(d|u_n), C_{g|n}(b|u_n)\right), \text{ with } P\left(q_j^U \ge r_j \ge q_j^L\right) = \eta, P\left(r_j \le q_j^U\right) = 0.5 + \frac{\eta}{2} \text{ and } P\left(r_j \le q_j^L\right) = 0.5 - \frac{\eta}{2} \text{ with } \eta = \alpha, \beta, \delta \text{ for } j = g, n, b; \text{ and where } a = 0.5 + \frac{\alpha}{2}, b = 0.5 + \frac{\beta}{2}, d = 0.5 - \frac{\alpha}{2} \text{ and } e = 0.5 - \frac{\beta}{2}. \Box$$

Proof: See Appendix.

Interestingly, copulas in Result 1 arise from a specific hierarchical dependence structure among green, neutral, and brown assets, shown in the upper panel of Figure 2. This dependence is given by a

¹¹ This modelling flexibility explains why this framework is the backbone of scenarios for stress testing. See for instance: shorturl.at/GHQV7.

C-vine copula,¹² where the central node in the first tree (T_1) represents neutral asset returns, and the edges connecting two nodes capture joint dependence between the returns of those nodes through bivariate copulas, allowing conditional dependence between those two variables to be computed. Likewise, the second tree (T_2) reflects two nodes representing green and brown asset returns conditional on neutral asset returns, with the edges providing information on the joint dependence between those variables as given by the corresponding copula. For the three bivariate copulas arising from this dependence structure, we can obtain all conditional copulas involved in Result 1 necessary to compute the probability of different climate transition scenarios.

[INSERT FIGURE 2 HERE]

Result 2. The joint density for the returns of a financial institution *i* and a disorderly transition scenario is:

$$f(r_{i}, r_{g} \ge q_{g}^{\beta}, r_{b} \le q_{b}^{\alpha}; r_{n}) = \int_{0}^{1} C(u_{b} \le \alpha, u_{g} \ge 1 - \beta | u_{n}, u_{i}) f_{i}(F_{i}^{-1}(u_{i})) du_{n},$$
(3)

where $C(u_b \le \alpha, u_g \ge 1 - \beta | u_n, u_i)$ is given by:

$$C_{b|i,n}(C_{b|n}(\alpha|u_n)|u_i) - C_{bg|i,n}(C_{b|n,n}(C_{b|n}(\alpha|u_n)|u_i), C_{g|i,n}(C_{g|n}(1-\beta|u_n)|u_i)).$$
 Swapping

around the green and brown subscripts, the density for the hothouse world scenario follows. As for the orderly transition scenarios, density is computed as:

$$f(r_{i}, q_{b}^{U} \ge r_{b} \ge q_{b}^{L}, q_{g}^{U} \ge r_{g} \ge q_{g}^{L}, q_{n}^{U} \ge r_{n} \ge q_{n}^{L})$$

$$= \int_{0.5 - \frac{\delta}{2}}^{0.5 + \frac{\delta}{2}} C(d \le u_{b} \le a, e \le u_{g} \le b | u_{n}, u_{i}) f_{i}(F_{i}^{-1}(u_{i})) du_{n},$$
(4)

where:

$$C(d \le u_b \le a, e \le u_g \le b | u_n, u_i) = C_{bg|n,i} (C_{b|n,i}(a|\{u_n, u_i\}), C_{g|n,i}(b|\{u_n, u_i\})) + C_{bg|n,i} (C_{b|n,i}(d|\{u_n, u_i\}), C_{g|n,i}(e|\{u_n, u_i\})) - C_{bg|n,i} (C_{b|n,i}(a|\{u_n, u_i\}), C_{b|n,i}(e|\{u_n, u_i\})) - C_{b|n,i} (C_{b|n,i}(a|\{u_n, u_i\})) - C_{b|n,i} (C_{b$$

¹² For an analysis of vine copulas, see Bedford and Cooke (2002); Kurowicka and Cooke(2006); Aas et al. (2009). In the trivariate case, the C- and D-vine copulas are equivalent when the pivotal node in the first tree of the C-vine is the central node in the first tree of the D-vine.

Proof: See Appendix.

Remarkably, in Result 2 the conditional copulas required to obtain the joint densities under different climate transition scenarios arise from a specific hierarchical dependence structure between the financial institution and market assets, represented in the lower (shaded) panel of Figure 2 through a C-vine copula. The first tree (\overline{T}_1) connects the returns of the financial firm with the two nodes of the second tree (T_2) of the hierarchical dependence of the assets in the market. For the three bivariate copulas arising from this dependence structure, we can obtain all the conditional copulas involved in Result 2.

From Results 1 and 2 we now can now compute the $CTER_i$ value. For a disorderly transition scenario,¹³ this is:

$$E\left(r_{i} \mid r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}; r_{n}\right)$$

$$= \frac{1}{P\left(r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}; r_{n}\right)} \int_{0}^{1} \int_{0}^{1} F_{i}^{-1}(u_{i}) C\left(u_{b} \leq \alpha, u_{g} \geq 1 - \beta \mid u_{n}, u_{i}\right) du_{n} du_{i},$$
(5)

while for the hothouse world scenario, the value of the $CTER_i$ is obtained by swapping around the green and brown subscripts. For an orderly transition scenario, $CTER_i$ is computed as:

$$E(r_{i}|q_{b}^{U} \ge r_{b} \ge q_{b}^{L}, q_{g}^{U} \ge r_{g} \ge q_{g}^{L}, q_{n}^{U} \ge r_{n} \ge q_{n}^{L})$$

$$= \frac{1}{A} \int_{0}^{1} \int_{0.5 - \frac{\delta}{2}}^{0.5 + \frac{\delta}{2}} F_{i}^{-1}(u_{i}) C(d \le u_{b} \le a, e \le u_{g} \le b|u_{n}, u_{i}) du_{n} du_{i}.$$
(6)

where $A = P(q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L, q_n^U \ge r_n \ge q_n^L)$. As for the values of $CTVaR_i^{\gamma}$ and $CTES_i^{\gamma}$, we need information on the joint probability of each climate transition scenario and the returns of firm *i*, and also information on the joint density between that scenario and those returns, given that returns for firm *i* are below a threshold given by $CTVaR_i^{\gamma}$. This information is described as follows.

Result 3. The probability of a disorderly transition scenario when returns for firm *i* are below a threshold given by $CTVaR_i^{\gamma}$ is:

¹³ Proofs of Eqs. (5) and (6) are reported in the Appendix.

$$P\left(r_{i} \leq CTVaR_{i}^{\gamma}, r_{b} \leq q_{b}^{\alpha}, r_{g} \leq q_{g}^{\beta}; r_{n}\right)$$

$$= \int_{0}^{F_{i}\left(CTVaR_{i}^{\gamma}\right)} \int_{0}^{1} C\left(u_{b} \leq \alpha, u_{g} \geq 1 - \beta \left|u_{n}, u_{i}\right) du_{n} du_{i}.$$
(7)

The probability for the hothouse world scenario follows by swapping around the green and brown subscripts. For an orderly transition scenario, this is:

$$P(r_{i} \leq CTVaR_{i}^{\gamma}, q_{b}^{U} \geq r_{b} \geq q_{b}^{L}, q_{g}^{U} \geq r_{g} \geq q_{g}^{L}, q_{n}^{U} \geq r_{n} \geq q_{n}^{L})$$

$$= \int_{0}^{F_{i}(CTVaR_{i}^{\gamma})} \int_{0.5 - \frac{\delta}{2}}^{0.5 + \frac{\delta}{2}} C(d \leq u_{b} \leq a, e \leq u_{g} \leq b | u_{n}, u_{i}) du_{n} du_{i}. \Box$$

$$(8)$$

Proof: See Appendix.

Result 4. The joint density for a disorderly transition scenario and the returns of a financial institution *i* when those returns are below a threshold $CTVaR_i^{\gamma}$ is:

$$f\left(r_{i} \leq F_{i}\left(CTVaR_{i}^{\gamma}\right), r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}; r_{n}\right) = \int_{0}^{F_{i}\left(CTVaR_{i}^{\gamma}\right)} f\left(r_{i}, r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}; r_{n}\right) du_{i}.$$
 (9)

The density for the hothouse world scenario follows by swapping around the green and brown subscripts. For an orderly transition scenario, this is:

$$f(r_{i} \leq CTVaR_{i}^{\gamma}, q_{b}^{U} \geq r_{b} \geq q_{b}^{L}, q_{g}^{U} \geq r_{g} \geq q_{g}^{L}, q_{n}^{U} \geq r_{n} \geq q_{n}^{L})$$

$$= \int_{0}^{F_{i}(CTVaR_{i}^{\gamma})} f(r_{i}, q_{b}^{U} \geq r_{b} \geq q_{b}^{L}, q_{g}^{U} \geq r_{g} \geq q_{g}^{L}, q_{n}^{U} \geq r_{n} \geq q_{n}^{L}) du_{i}.\Box$$

$$(10)$$

Proof: See Appendix.

We can now obtain the value of $CTVaR_i^{\gamma}$ under different transition scenarios. For a disorderly transition scenario, the $CTVaR_i^{\gamma}$ is the quantile that verifies that $P\left(r_i \leq CTVaR_i^{\gamma} | r_g \geq q_g^{\beta}, r_b \leq q_b^{\alpha}; r_n\right) = \gamma$, namely:

$$\frac{P\left(r_{i} \leq CTVaR_{i}^{\gamma}, r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}; r_{n}\right)}{P\left(r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}; r_{n}\right)} = \gamma,$$

where the ratio of probabilities, denoted by $G(\cdot)$, derives from Result 1 and 3. Hence, given that $P(r_i \leq CTVaR_i^{\gamma}) = F_i(CTVaR_i^{\gamma})$, the value of $CTVaR_i^{\gamma}$ is computed as:¹⁴

$$CTVaR_i^{\gamma} = F_i^{-1} \big(G^{-1}(\gamma) \big). \tag{11}$$

For the hothouse world and orderly transition scenarios, the value of $CTVaR_i^{\gamma}$ is given by Eq. (11) but the ratio of probabilities is given by the corresponding $G(\cdot)$.

Finally, the tail risk effects from a disorderly transition scenario can be quantified with the $CTES_i^{\gamma}$ as:

$$E\left(r_{i} \mid r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}, r_{i} \leq CTVaR_{i}^{\gamma}; r_{n}\right) = \frac{1}{P\left(r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}, r_{i} \leq CTVaR_{i}^{\gamma}; r_{n}\right)}$$
$$\int_{0}^{F_{i}\left(CTVaR_{i}^{\gamma}\right)} \int_{0}^{1} F_{i}^{-1}(u_{i}) C\left(u_{b} \leq \alpha, u_{g} \geq 1 - \beta \mid u_{n}, u_{i}\right) du_{n} du_{i},$$
(12)

where $P(r_g \ge q_g^{\beta}, r_b \le q_b^{\alpha}, r_i \le CTVaR_i^{\gamma}; r_n)$ is given by Result 3. Swapping around the green and brown subscripts we obtain $CTES_i^{\gamma}$ for the hothouse world transition scenario. For an orderly transition scenario, $CTES_i^{\gamma}$ is given by:

$$E(r_{i}|r_{i} \leq CTVaR_{i}^{\gamma}, q_{b}^{U} \geq r_{b} \geq q_{b}^{L}, q_{g}^{U} \geq r_{g} \geq q_{g}^{L}, q_{n}^{U} \geq r_{n} \geq q_{n}^{L})$$

$$= \frac{1}{B} \int_{-\infty}^{F_{i}(CTVaR_{i}^{\gamma})} \int_{0.5-\frac{\delta}{2}}^{0.5+\frac{\delta}{2}} F_{i}^{-1}(u_{i})C(d \leq u_{b} \leq a, e \leq u_{g} \leq b|u_{n}, u_{i})du_{n} du_{i}, \quad (13)$$

where $B = P\left(r_i \leq CTVaR_i^{\gamma}, q_b^U \geq r_b \geq q_b^L, q_g^U \geq r_g \geq q_g^L, q_n^U \geq r_n \geq q_n^L\right)$.

2.4 Estimation

Estimation of the systemic impact of climate transition scenarios requires information on the copula functions as represented in Figure 2. Those copulas are estimated using the two-step inference functions for margins (IFM) approach (Joe and Xu, 1996).

In the first IFM step, we estimate the univariate marginal distribution functions of the j = i, g, n, breturns by maximum likelihood (ML), where the dynamics of those returns is assumed to be described by an autoregressive moving average (ARMA) model of order *m* and *k*:

¹⁴ Proofs of Eqs. (11)-(13) are reported in the Appendix.

$$r_{j,t} = \phi_0 + \sum_{q=1}^{m} \phi_q r_{j,t-q} + \sum_{k=1}^{k} \varphi_k \epsilon_{j,t-k} + \epsilon_{j,t} , \qquad (14)$$

where ϕ_q and φ_r denote the parameters of the AR and MA components of the model, and $\epsilon_{j,t}$ is the stochastic component that is assumed to have a zero mean and variance with dynamic behaviour as represented by a threshold generalized autoregressive conditional heteroskedasticity (TGARCH) model:

$$\sigma_{j,t}^{2} = \omega_{0} + \sum_{k=1}^{K} \beta_{q} \sigma_{j,t-k}^{2} + \sum_{h=1}^{H} \alpha_{h} \epsilon_{j,t-h}^{2} + \sum_{h=1}^{H} \delta_{h} \mathbf{1}_{t-h} \epsilon_{j,t-h}^{2},$$
(15)

where ω_0 , β_q and α_h are the parameters of the volatility model, and where $1_{t-h} = 1$ if $\epsilon_{j,t-h} < 0$, and otherwise is zero. The parameter δ_h accounts for the asymmetric effect of shocks, thus, for $\delta_h > 0$, negative shocks have more impact on variance than positive shocks. Asymmetries and fat tails in the marginal distribution of returns are captured by assuming that the return distribution is given by Hansen's (1994) skewed-t density with ϑ ($2 < \vartheta < \infty$) degrees of freedom and asymmetry parameter λ ($-1 < \lambda < 1$). The number of lags for the mean and variance of returns is selected using the Akaike information criterion (AIC). From marginal models, we obtain the pseudo-sample observations for the copula as given by the integral probability transformation of standardized returns.

In the second IFM step, the multivariate dependence is captured through a hierarchical estimation of the time-varying copula parameters. First, we estimate the parameters of the dependence model for the green, neutral, and brown assets as represented in the upper panel of Figure 2 (common for all financial institutions), using sequential ML (Aas et al. 2009; Hobaek Haff, 2013), which consists of estimating bivariate copula parameters for the first tree level using the probability integral transformations from marginals as pseudo-sample observations, and then obtaining pseudo-sample observations from those copulas to estimate copula parameters for the next tree. Second, for each financial institution *i* we estimate the dependence structure of that financial institution with the market assets as represented in the lower panel of Figure 2. Bivariate copula parameters are estimated using sequential ML: copulas for the first tree are estimated using both pseudo-observations from the second tree of the dependence model (the conditional copula values for green and brown assets) and pseudo-sample observations from the probability integral transformation of the marginal of returns for financial firm *i*, and then, from the bivariate copulas in the first tree we generate pseudo-observation to estimate the parameters of the copula in the second tree.

For estimation of all the bivariate dependencies represented in Figure 2, we use different bivariate copula specifications with time-varying parameters following Patton (2006) as reported in Table 2, selecting the most appropriate copula model using the AIC corrected for small sample bias (Breymann et al., 2003).

[INSERT TABLE 2 HERE]

3. Data

3.1 Firm vulnerability to climate transition risk

To delimit climate transition scenarios, we need to categorize green, brown, and neutral asset returns. To that end, we use rated information on the vulnerability of the firm's value to transition risk as reported by Sustainalytics – a widely recognized leading provider of environmental, social, and governance (ESG) information.

On an annual basis, Sustainalytics computes a rating called the carbon risk score (CRS), which is based on exposure to and management of carbon transition risk by firms in 146 subindustries. Carbon exposure, which largely depends on the type of business, measures the extent to which carbon risk is materialized across the firm's value chain (including operations, products, and services). It is measured by subindustry and is specifically adjusted at the firm level by considering (a) company operations or product mix deviations with respect to its subindustry, and (b) the firm's financial strength and geographical components that could undermine the firm's capacity to address carbon risks. Management of carbon risk measures the firm's management ability and quality in terms of reducing emissions and related carbon risks. Management, as characterized by implementation of company's policies, programmes and systems in operations, products, and services, is ultimately reflected in (a) reductions in carbon emissions, (b) level of reliance on fossil fuels, and (c) development of greener products and services. Once carbon risk management is accounted for, the remaining risk is unmanaged carbon risk, defined as unmanageable risks beyond the control of the company and manageable risks that have not been accounted for.

For unmanaged carbon risk, Sustainalytics assigns a CRS that evaluates the extent to which the company's value is placed at risk by transition to a low-carbon economy. Accordingly, firms are rated

with a CRS between 0 and 100, reflecting negligible (0), low (1 to 9.99), medium (10 to 29.99), high (30 to 49.99), and severe (50 or more) carbon transition risk.¹⁵ As a transition risk measure, the CRS metric is more informative than carbon emissions according to Greenhouse Gas (GHG) Protocol Scopes 1, 2, and 3, as it considers not only carbon emissions information, but also policies and actions to manage the impact of transition to a low-carbon economy on a firm's value.¹⁶ Moreover, information on CRS ratings is available to institutional and private investors, who can assess the resilience of their investments to climate transition risks (Reboredo and Otero, 2021; Reboredo and Ugolini, 2022).

Using firm-level CRS values, we sort firms into quintiles in such a way that they are categorized as green or brown when included in the first and fifth quintiles, respectively, and as neutral otherwise. The distinctive feature of green, neutral, and brown firms is their vulnerability to transition to a low-carbon economy, with green (brown) firms exhibiting the lowest (highest) risk exposure, and neutral firms having average risk exposure. Using returns for non-financial firms within each category, we compute green, neutral, and brown returns as the average returns for the companies included in the corresponding category.¹⁷

3.2 Data source

Our dataset includes both European financial firms and European listed firms that are annually rated with a CRS. The sample goes from 2013, when information on CRS at the firm level becomes available, to 2020, with all data sourced from Bloomberg.

The sample includes 939 European listed firms, representing 99.4% of the firms included in the Eurostoxx-600 index and 97% of market capitalization of that index at the end of 2020. Those firms are annually grouped into the green, neutral, or brown asset categories, depending on whether they belong to the first CRS quintile, to the second, third or fourth CRS quintiles, or to the fifth CRS quintile,

¹⁵ For a detailed analysis of rating methods, see: <u>https://www.morningstar.com/lp/low-carbon-economy</u>.

¹⁶ We have performed a robustness check by considering green, brown and neutral portfolios using information from scope 1 and scope 2 CO2 carbon emissions at the firm level. Empirical evidence, available on request, lead to similar qualitative results presented here.

¹⁷ Alternatively, we could also use market weights to determine the returns of each asset category, even though the dynamics of the returns for each category might be mainly determined by a single firm with large market capitalization.

respectively. Weekly returns for each asset class are computed as the average of the log-returns of the assets in the corresponding category.

The sample of European financial firms includes 190 firms representing 85% of the Euro Stoxx financials index (data for the end of 2020): 43 banks (24 of which are classified as domestic systemically important banks by the Financial Stability Board in 2020), 36 insurance companies, 52 financial services companies, and 59 real estate firms. We consider various categories of financial firms given that their different business models are likely to affect their exposure to climate transition risks. Systemic risk for similar financial firms has been investigated by Engle et al. (2015) and for a similar set of banks by Borri and Giorgio (2022). By market capitalization (data for the end of 2020), the largest firms are as follows: HSBC, BNP Paribas, Banco Santander, and Intesa Sanpaolo (banks); Allianz, Chubb, Zurich Insurance, and AXA (insurance companies); UBS Group, London Stock Exchange, Deutsche Böerse, and Credit Suisse (financial services firms); and Deutsche Wohnen, Segro, Gecina, and LEG Immobilien (real estate firms). Total capitalization is 1,680 billion euros, for a median value of 10 billion euros. For all the financial firms, we compile data for weekly market prices in euros, and use data on debt book value and the market value of the equity in euros obtained from Compustat.

Table 3 presents summary statistics for the returns of different asset and financial firm categories. It confirms that green, neutral, and brown assets have dissimilar performances in terms of returns and volatilities, with green assets outperforming brown and neutral assets in terms of greater realized returns and lower volatility. Moreover, probability distributions of green, neutral, and brown assets also differ according to skewness and kurtosis information, and according to tail behaviour as reflected in the empirical value-at-risk (VaR) and expected shortfall (ES) values in the left and right sides of the distribution. Extreme movements in the green, neutral, and brown returns are dissimilar, with brown assets experiencing larger extreme downward movements than green assets.

For the financial sample, Table 3 shows that financial services companies outperform the other categories, while real estate and insurance companies have similar average returns. Banks yield average negative returns and display greater volatility than the other financial firms. All financial firms are characterized by higher volatility than market assets and also by negative skewness and fat tails.

According to the empirical VaR and ES metrics, banks show higher levels of tail risk than the other financial firms.

[INSERT TABLE 3 HERE]

Figure 3 shows the cumulative performance of green, neutral, and brown assets, along with the (average) cumulative returns for each financial institution category. Over the sample period, cumulative returns for green assets are above the cumulative returns for brown assets, although the differences are slightly reduced in the last year of the sample period due to the COVID-19 pandemic. Financial firms show different patterns, with banks underperforming the other financial firms and experiencing severe cuts between mid-2015/mid-2016 and from the pandemic outset. Financial services and real estate returns display similar dynamics, closely co-moving with neutral asset returns.

[INSERT FIGURE 3 HERE]

3.3 Financial firm exposure to green, neutral, and brown assets

To assess average exposure of financial firms to green, neutral, and brown assets, for each financial firm we run a capital asset pricing model (CAPM)-type regression, where the market return factor is decomposed into green, neutral, and brown asset returns, while the three regression betas provide information on the sensitivity of each financial firm's returns to the different asset returns. The product of those betas multiplied by the respective average values of the green, neutral, and brown asset returns under specific climate transition scenarios yields the average impact of a particular scenario on a financial institution. We assess those average impacts in three circumstances, as follows: (a) green and brown returns are above and below their respective median values, reflecting a disorderly transition scenario using median quantiles as thresholds; (b) brown and green returns are above and below their respective median values, reflecting a hothouse world scenario using medians as thresholds; and (c) green, neutral, and brown returns are below their 75% and above their 25% respective quantiles, consistent with an orderly transition scenario.

Panel A of Figure 4 shows the distribution of betas across the financial firms included in different categories. The graphic evidence indicates that banks and insurance companies are more exposed to brown than to green asset returns, whereas financial services and real estate firms are more sensitive to

green and neutral asset returns than to brown assets. Banks overall show the highest average beta for brown returns. There is also wide dispersion in the betas within each financial firm category, with the betas for neutral assets exhibiting the greatest dispersion.

[INSERT FIGURE 4 HERE]

Consistent with the distribution of betas, the distribution of average impacts from different climate transition scenarios differs widely across and within different categories of financial firms, as reflected in Panel B of Figure 4. Specifically, banks receive the highest positive and lowest positive average return impacts from a hothouse world scenario and a disorderly transition scenario, respectively, whereas the opposite occurs for real estate firms. The average impact for insurance firms is similar for the different transition scenarios, while for financial services, the impact of a disorderly transition scenario is slightly more positive than of a hothouse world scenario. Finally, graphically reflected is great heterogeneity in the size of the impact within and between climate transition scenarios.

4. Empirical evidence on the systemic impact of climate transition

4.1 Model estimation

We start by estimating marginal model parameters for green, neutral, and brown asset returns and for each financial firm in our sample. Table 4 reports estimates, where the number of lags in the mean and variance specifications are the values that minimize the AIC, considering different values between 0 and 2. Evidence for green, neutral, and brown marginal densities reported in the first three columns of Table 4 shows that those returns exhibited no serial dependence, whereas conditional variances were persistent and displayed significant positive leverage effects, with bad news having a greater impact on volatility than good news. The distribution of green, neutral, and brown assets is negatively skewed and has fat tails. Goodness-of-fit metrics for the model residuals point to the fact that no serial correlation remains, in either the residuals in levels or the residuals squared, and that the skewed-t distribution adequately accounts for the asymmetry and tail return features, given that the Kolmogorov-Smirnov (KS) test supports uniformity in the standardized model residuals.

[INSERT TABLE 4 HERE]

As the number of marginal models for the financial firms is large, rather than individual parameter estimates and goodness-of-fit results, we report only summary statistics for firms grouped into the four categories reported in the last four columns of Table 4. Overall, some financial firms show evidence of serial correlation in returns, volatility is persistent (mainly for banks), and there is some evidence of positive leverage effects for financial firms that is smaller in size than for market assets. Goodness-offit tests support the fitted marginal models, reporting no misspecification errors for any of the financial firms, and confirming that the return distribution is well characterized by a skewed student-t with fat tails, which, in some cases, behaves as a symmetric student-t.

Using the probability integral transformation obtained from marginal model estimations, we first estimate the market dependence structure (see the upper panel of Figure 2). Table 5 shows parameter estimates for the three estimated copulas that describe the dependence structure for green, neutral, and brown assets. The copula that best characterizes dependence between green and neutral assets is a static BB1 copula with average positive dependence and asymmetric tail dependence (greater lower tail dependence). Dependence between brown and neutral assets is also well described by a BB1 copula, with positive dependence oscillating over time, basically influenced by one of the copula parameters. Finally, conditional dependence between green and brown assets is well characterized by an independent copula.

[INSERT TABLE 5 HERE]

Table 6 summarizes estimates of the dependence structure between financial firms and the market (see the lower (shaded) panel of Figure 2). Copula estimates indicate that dependence between financial institutions and green returns is positive for most financial firms, with some evidence of independence for 22.2% of firms. Likewise, dependence between financial firms and brown returns conditional on neutral asset returns is mostly positive and low, with evidence of independence for 31.1% of firms. Consistent with the market dependence information, green and brown returns conditional on neutral and financial firm returns are independent.

[INSERT TABLE 6 HERE]

4.2 Evidence on the systemic risk impacts of climate transition scenarios

Using information from the bivariate copulas that characterize the market and the dependence structure for financial firms, for the sample period we compute the systemic risk impact for each financial firm arising from each climate transition scenario (hothouse world, disorderly transition, and orderly transition). Specifically, at each time *t* we compute the values for the systemic metrics, i.e., CTER, CTVaR, and CTES, for confidence levels of $\alpha = 0.20$, $\beta = 0.20$, and $\gamma = 0.10$, and for quantiles $F_i(q_i^U) = 0.60$ and $F_i(q_i^L) = 0.40$ for j = g, n, b.¹⁸

4.2.1 Systemic risk impacts of climate transition scenarios

Figure 5 depicts estimates of the three systemic risk metrics for the different types of financial institutions. As CTER is an additive measure, aggregated values for each financial institution type are obtained as the weighted average of the individual values, weighted by the market value of each firm over the total market value of all financial firms in that category. Since CTVaR and CTES are not additive measures, for each climate transition scenario we report median values in the cross-section sample, along with 25% and 75% percentile values (represented by shaded areas).

Figure 5 highlights dissimilar temporal dynamics patterns of the systemic risk impact of climate transition scenarios for different types of financial firms. For the CTER metric, Panel A of Figure 5 shows that the value of all financial firms deteriorates in a hothouse world scenario. However, the decline in CTER from a hothouse world scenario is of a smaller size for banks (average value of -0.8%, receiving positive impacts at specific time periods), while real estate firms receive the largest impact (average value of -2.3%). This evidence is consistent with the diverse exposure of financial firms to different type of assets, with banks more exposed to brown asset than real estate firms (see Figure 4). The impact of a hothouse world scenario is therefore more severe for real estate firms than for banks. In contrast, real estate firms and financial services firms are positively affected by a disorderly transition, while banks receive a negative impact and insurance companies a slightly positive impact. Not surprisingly, the effects of the COVID-19 pandemic are reflected in all the transition scenarios, even

¹⁸ Those confidence levels correspond to empirical quantiles for green, neutral, and brown weekly returns, respectively, as follows: $q_{0.2} = -1.27$, -1.44, -1.99; $q_{0.4}^L = 0.21$, -0.07, -0.19; $q_{0.6}^U = 0.87$, 0.76, 0.75; $q_{0.8} = 1.66$, 1.58, 1.89. The online appendix provides a robustness check for different confident levels.

despite the systemic impact from a disorderly transition scenario being higher for banks than for the remaining firms. For all financial firms, the impact of an orderly transition scenario is negligible.

[INSERT FIGURE 5 HERE]

Interestingly, median values and interquartile ranges for CTVaR represented in Panel B of Figure 5 also reflect the greater exposure of banks to brown assets, as CTVaR estimates for banks in a hothouse world scenario are higher (average value of -4.8%) than in a disorderly transition scenario (average value of -7.2%); this is due to the fact that banks are more positively impacted by upturns in brown asset prices than by upturns in green asset prices. Remarkably, the opposite is observed for insurance, financial services, and real estate firms, where the value of CTVaR is higher in the disorderly transition scenario than in the hothouse world scenario (e.g., for real estate firms, average CTVaR values are -2.2% and -5.5% in the former and latter scenarios, respectively). Also, the non-banking sector presents higher cross-section heterogeneity than the banking sector, especially in the hothouse world scenario.

Panel C of Figure 5 shows that expected tail losses for insurance, financial services, and real estate firms are larger in a hothouse world scenario that in a disorderly transition scenarios, and that the opposite occurs for banks. The temporal dynamics of the median CTES is similar to the dynamics of CTVaR, with abrupt downward movement in the COVID-19 period.

All in all, evidence on the impact of different climate transition scenarios for different types of financial firms are, not surprisingly, consistent with the degree of exposure of those institution to changes in green and brown asset prices.

Table 7 presents descriptive statistics for the three risk metrics under different scenarios, considering the whole set and different categories of financial firms as presented in Figures 4 and 5. Descriptive results confirm the above-described graphical evidence.

[INSERT TABLE 7 HERE]

4.2.2 Systemic risk effects of climate transition scenarios for individual firms

Table 8 presents average values over the sample period for the three systemic risk measures in the three climate transition scenarios for the four largest institutions within each category. The evidence

in Table 8, consistent with the graphical evidence reported in Figure 4, is that financial institutions are diverse in terms of the impact of different scenarios.

Regarding the banking sector, the CTER, CTVaR, and CTES systemic risk metrics point to improved performance in a disorderly transition scenario and deteriorated performance in a hothouse world scenario for the two largest banks, HSBC and BNP Paribas. In contrast, the systemic risk metrics for Santander and Intesa Sanpaolo, more exposed to brown than to green assets, deteriorate more in a disorderly transition scenario than in a hothouse world scenario. This empirical evidence confirms that banks differ widely in terms of their exposure to climate risk,¹⁹ a fact that needs to be borne in mind in any regulation regarding that risk.

For the four largest insurance firms, the evidence indicates that CTER, CTVaR, and CTES average values are better for Alliance, Chubb and AXA in a disorderly transition scenario compared to a hothouse world scenario, whereas the impact on Zurich of any of the three climate transition scenarios is fairly similar.

Finally, for the largest firms within the financial services and real estate categories, average CTER, CTVaR, and CTES values confirm enhanced performance in a disorderly transition scenario compared to a hothouse world scenario. This finding, corroborating the evidence for the financial services and real estate firms overall, as presented in Figures 4 and 6, suggests that those firms, on the whole, are well positioned for transition to a low-carbon economy in which green (brown) firms would be revalued upwards (downwards).

[INSERT TABLE 8 HERE]

4.2.3 Systemic risk impact of climate transition scenarios for individual countries

To explore the systemic risk of climate transition scenarios for individual countries, for CTER (additive) we compute average values for each financial institution in each country and aggregate those values using, as weights, the market value of each firm over the total market value of all financial firms in the corresponding country. For CTVaR and CTES (non-additive), we obtain the average value for each

¹⁹ For example, the French banking sector holds a higher share of loans to low emitters than Spanish or Italian banking sectors (Alogoskoufis et al., 2021).

financial firm in each country over the sample period and take the median values of the averages as indicative of the VaR and ES.

Figure 6 depicts the systemic impact of the three climate transition scenarios on the European countries included in our sample. In a disorderly transition scenario, the financial systems of Finland, France, and Norway benefit, given that their financial firms show higher (lower) exposure to green (brown) than to brown (green) assets. More specifically, average CTER values are higher than in other countries and CTVaR and CTES values also indicate better tail risk performance. In contrast, the financial systems of Ireland, Portugal, Poland and Spain are the countries most exposed to a disorderly transition, displaying the poorest performance for CTER, and also for tail risk, which is particularly high for the Italian financial system. Overall, most European countries show vulnerability to a disorderly transition scenario.

[INSERT FIGURE 6 HERE]

Regarding the hothouse world scenario, Figure 6 indicates that the financial systems of Portugal, Ireland, Luxembourg, and Spain would benefit, increasing their average CTER and reducing tail risk in terms of CTVaR and CTES with respect to the other European countries. In contrast, the financial systems of Finland, France, and Norway show a poorer profile in terms of average returns and tail risk metrics. It is important to remark that, although the southern European financial sector incurs fewer losses in the hothouse scenario, this scenario would trigger harmful physical climate events in the Mediterranean countries.²⁰

Finally, for an orderly transition scenario, the evidence points to the financial systems of Ireland and Portugal as the poorest performers in terms of CTER and also in terms of tail risk, while the best performers in terms of tail risk are the financial systems of Finland, Switzerland, and Belgium.

Taken together, the overall picture of climate transition risk across European financial systems is very diverse, with countries ranking differently depending on the climate transition scenario. This result

²⁰ Analysis of physical risk is out-of-scope in this study. This remark must be done to highlight that it is necessary to take into account a comprehensive set of measures and indicators to capture the overall implication of climate risk. See ACPR and Banque de France (2021) and Alogoskoufis et al. (2021).

might be a consequence of loan portfolios that are different depending on the country where the financial institution is located.

5. Capital implications of climate transition risk

To assess the implication of each climate transition scenario in terms of capital shortfalls for financial firms, following Brownlees and Engle (2017), we define the climate transition capital shortfall (CTCS) for a financial institution i at time t as:

$$CTCS_{i,t} = kD_{i,t} - (1-k)(1 + LRCTER_{i,t})W_{i,t},$$
(16)

where $LRCTER_{i,t} = \exp(52 \cdot CTER_{i,t}) - 1$ is the one-year-ahead climate transition expected returns, representing the expected change in equity under a specific climate transition stress scenario (e.g., computed as per Eq. (5) for a disorderly transition scenario), *k* is the fraction of assets that the financial firm has to reserve in the case of a crisis (the prudential capital ratio), $D_{i,t}$ is the debt book value, and $W_{i,t}$ is the equity market value. The CTCS, given by the difference between the required and available capital, is a forward-looking metric, as it relies on expected change in the market value of financial institution *i*. The dynamics of the CTCS is not only influenced by the impact on returns of the climate transition scenario, as given by the CTER, but also by the dynamics of market capitalization and debt. From the CTCS, we can define the climate transition systemic capital shortfall (CTRISK) for a financial institution *i* as a positive capital shortfall value:

$$CTRISK_{i,t} = \max\{0, CTCS_{i,t}\}.$$
(17)

Using information on the debt book value, market capitalization for each financial firm (sourced from Compustat), and CTER values (as reported in the previous section and expressed on an annual basis), we compute CTCS and CTRISK values for the different climate transition scenarios, considering a capital ratio of k = 5.5%.²¹ Figure 7 represents the dynamics of the total CTRISK value for the four most impacted firms and the remaining firms within each category, showing that capital shortfall differs across financial firms and over time.

²¹ This ratio, also used by Engle et al. (2015) for European financial firms, ensures no capital shortfall for a leverage of 18.2.

[INSERT FIGURE 7 HERE]

Banks experience substantial capital shortfalls from a disorderly transition scenario, at average values of about 40 billion euros, peaking at 120 billion euros during a high-risk period (such as a COVID-19-like pandemic). Substantial differences exist between banks, with the four most impacted banks accounting for a small fraction of the total capital shortfall. For those banks, Table 9 presents average CTRISK values, indicating that, in a disorderly transition scenario, the most impacted banks, excepting Santander, experience average capital shortfalls that represent an important fraction of their market capitalization. In contrast, in a hothouse world scenario, and even though average values for total CTRISK are quite similar to those for a disorderly transition scenario, there is great dispersion between banks, with the most four impacted banks accounting for a large fraction of the total CTRISK value primarily Credit Agricole (average CTRISK value representing 65% of its market capitalization). The capital impact of an orderly transition scenario is moderate, with average values of around 5 billion euros, and is concentrated in the most affected banks, with capital shortfalls representing a small fraction of their market capitalization. Overall, the empirical estimates point to a relatively manageable impact on bank capital of climate transition – in comparison with a financial crisis, when capital consumption is substantially greater; see, e.g., Engle et al. (2015) who report an average capital shortfall in a financial crisis of around 400 billion euros for European banks. The effects of climate transition in terms of positive capital shortfalls are concentrated in a small number of entities, and interestingly, as the average CTCS value is below zero, those positive capital shortfalls are absorbable by the banking sector.²²

[INSERT TABLE 9 HERE]

For insurance companies, capital shortfall estimates, depicted in Figure 7, show that these are barely affected in the orderly and disorderly transition scenarios, except during the COVID-19 pandemic, when capital shortfall peaks at 1 billion euros. However, capital shortfall is mostly affected in a hothouse world scenario. Table 9 evidences that capital losses for insurance firms are concentrated

 $^{^{22}}$ As the CTCS is negative on average, if the regulator allows for netting within the financial system, i.e. bail-in mechanism, the losses in a financial firm could be offset with the profits in the remainder financial institutions. This would be equivalent to avoid the use of max(...) function when assessing the CTRISK in Eq. (17).

in a small number of firms and are mainly affected by the hothouse world scenario, but overall representing a small percentage of their market capitalization.

Regarding financial services, Figure 7 shows that those firms are particularly affected in a hothouse world scenario, with average capital shortfall over the sample period of 15 billion euros; this figure is reduced by half in a disorderly transition scenario, and shrinks to less than 1 billion euros in an orderly transition scenario. As for insurance firms, Table 9 indicates that capital shortfalls for the most impacted financial service firms represent a small fraction of their market capitalization, with the exception of Deutsche Bank AG in the disorderly transition scenario.

Finally, Figure 7 shows that capital shortfalls for real estate firms are negligible in the disorderly and orderly transition scenarios, and although capital shortfalls are larger in a hothouse world overall, as reported in Table 9, they represent a small fraction of firm capitalization. This evidence is consistent with the greater unfavourable impact on real estate firms of a hothouse world scenario.

7. Conclusions

Moving towards a greener economy involves risks for the value of financial assets, with repricing effects (Carney, 2016) potentially having an impact on the stability of financial systems. In this paper, we address how climate transition risk, through its effects on asset prices, could impact financial stability. To that end, we characterize the behaviour of financial firm returns conditional on the dynamics of market returns for green, neutral, and brown assets (reflecting low, neutral, and high vulnerability, respectively) in the transition to a low-carbon economy. We consider three climate transition scenarios aligned with the narrative of main climate transition scenarios suggested by regulators and supervisory authorities , namely, disorderly transition, orderly transition, and hothouse world (no transition), featured in terms of relative changes in green, neutral, and brown asset prices arising from disrupted business models due to changes in the timing and speed of the adjustment towards a low-carbon economy. We then assess the systemic risk impact of those scenarios on financial firms in terms of the average return (climate transition expected returns), the minimum returns with some confidence level (climate transition value-at-risk), and the average return below that minimum threshold (climate

transition expected shortfalls), accounting for average and tail effects from transition scenarios for the value of financial firms.

We apply our methodology to European financial firms (banks, insurance companies, financial services companies, and real estate firms) over the period 2013-2020. Our main findings are that the systemic impact of climate transition scenarios varies widely across financial institutions. Banks experience more systemic impacts in the disorderly transition scenario than in the hothouse world scenario, while the opposite occurs for the other financial firm types, but especially for real estate firms. We also find that the systemic impact of the different climate transition scenarios is widely divergent within financial firm types, yielding potential winners and losers.

We also assess the implications of climate-related systemic risk in terms of capital shortfalls. For banks, capital shortfalls are negligible in the orderly transition scenario; however, in the disorderly transition and hothouse world scenarios, capital shortfalls are sizeable, although concentrated in a small number of entities and absorbable within the banking sector. For the remaining financial firms, we find that insurance firms experience small capital shortfalls in any climate transition risk scenario, whereas financial services and real estate firms experience modest capital losses in a hothouse world scenario, and negligible capital losses in the remaining scenarios.

The implications of this study go beyond risk management, as it provides a useful methodology for generating stress test scenarios for climate risk. Regulatory and supervisory authorities might also find in this study a flexible tool for evaluating the performance of financial firms under different distress scenarios coherent with the transition to a low-carbon economy, taking into account financial fears in the market through nonlinearities and tail dependencies.

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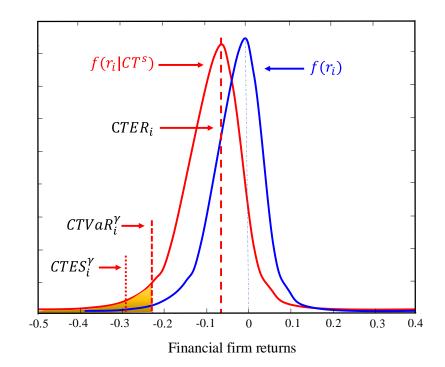
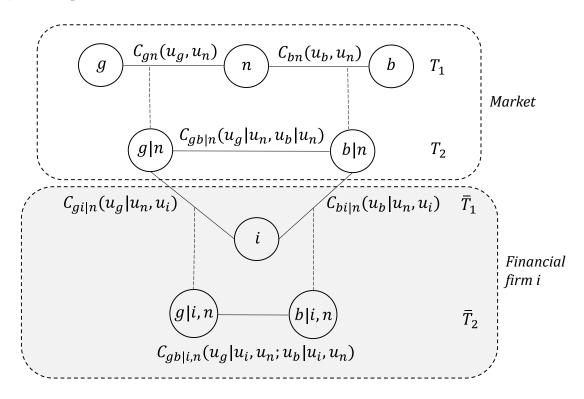


Figure 1. Systemic climate transition risk metrics: $CTER_i$, $CVaR_i^{\gamma}$, and $CTES_i^{\gamma}$.

Figure 2. Dependence structure between market and financial firm returns.



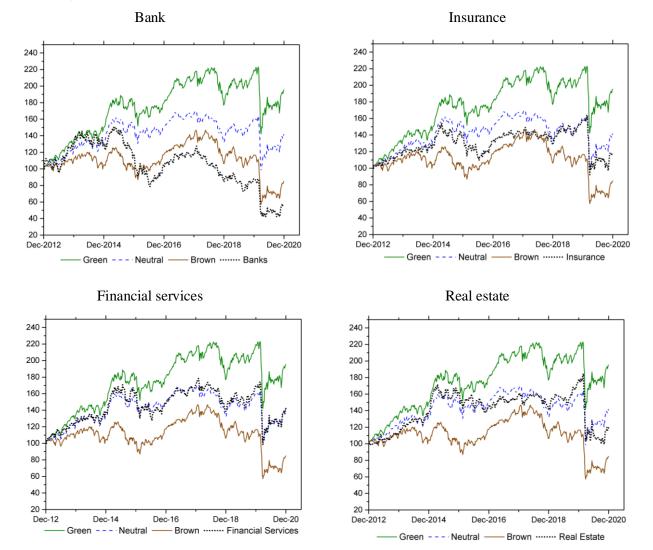
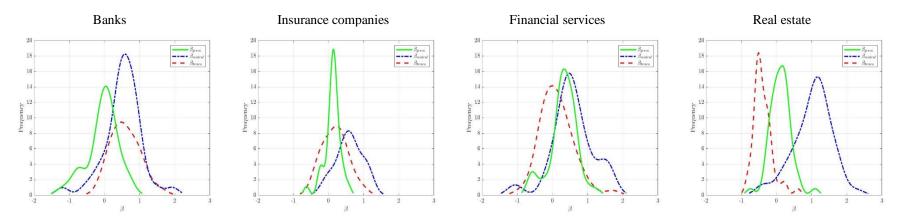


Figure 3. Cumulative returns for different asset classes and financial institutions.

Figure 4. Exposure of financial firms to green, neutral, and brown asset returns.

Panel A. Distribution of beta values for green, neutral, and brown asset returns.



Panel B. Distribution of average return impacts under different climate transition scenarios.

25

20

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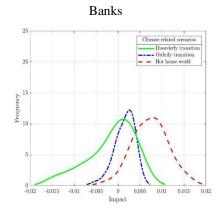
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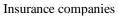
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0.005 0.01 0.015 0.02

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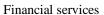


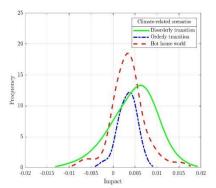
Climate-related scenarios

---- Orderly transition

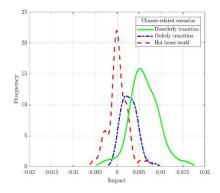
Hot house world

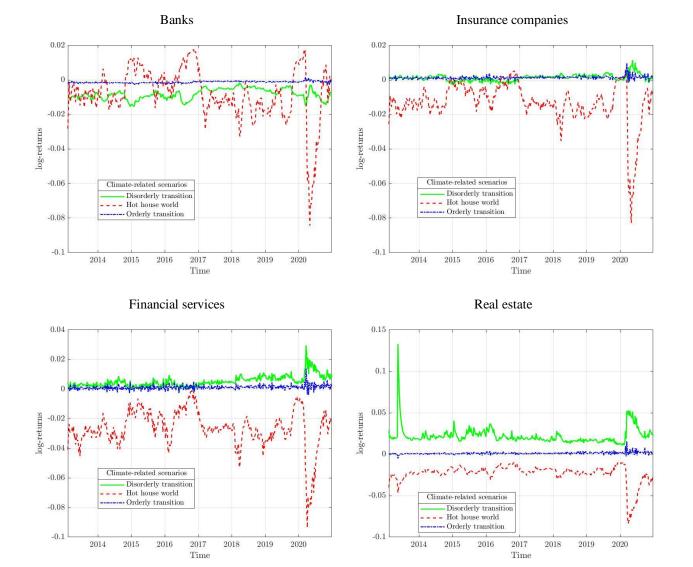
Disorderly transit







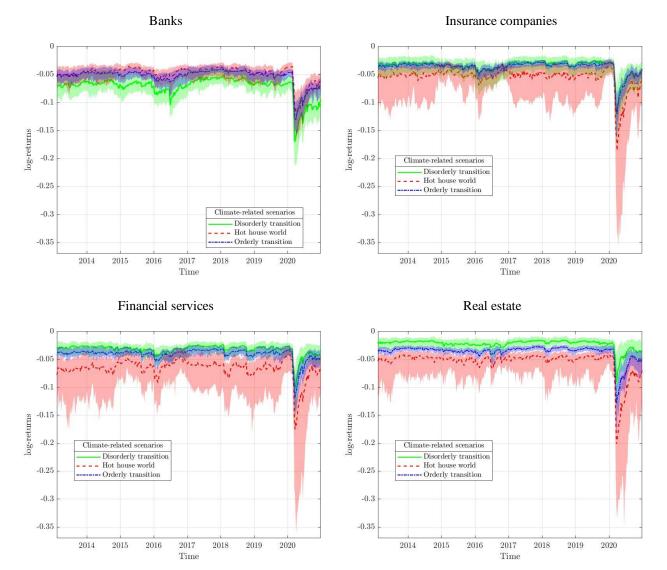




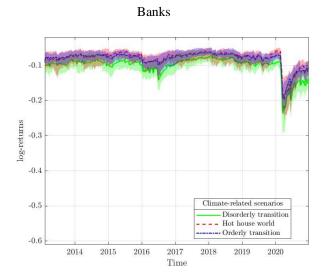
Panel A. CTER

Figure 5. Systemic risk of climate transition scenarios for different financial firm types.

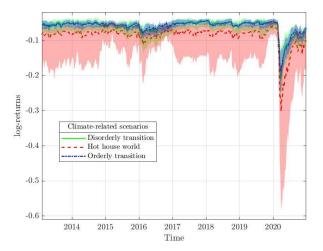
Panel B. CTVaR



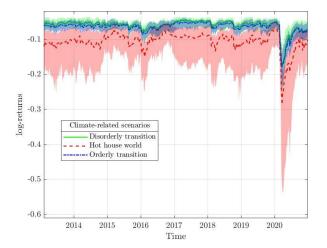
Panel C. CTES



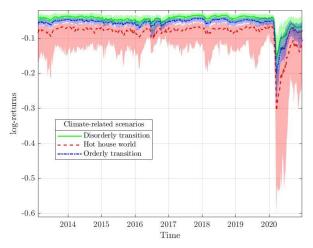
Insurance companies



Financial services







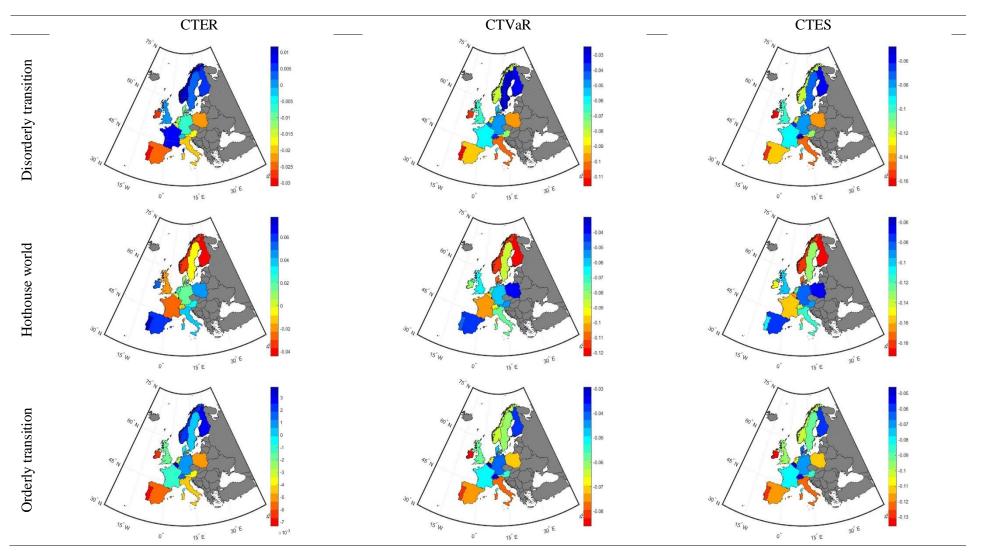


Figure 6. Systemic risk of climate transition scenarios by country.

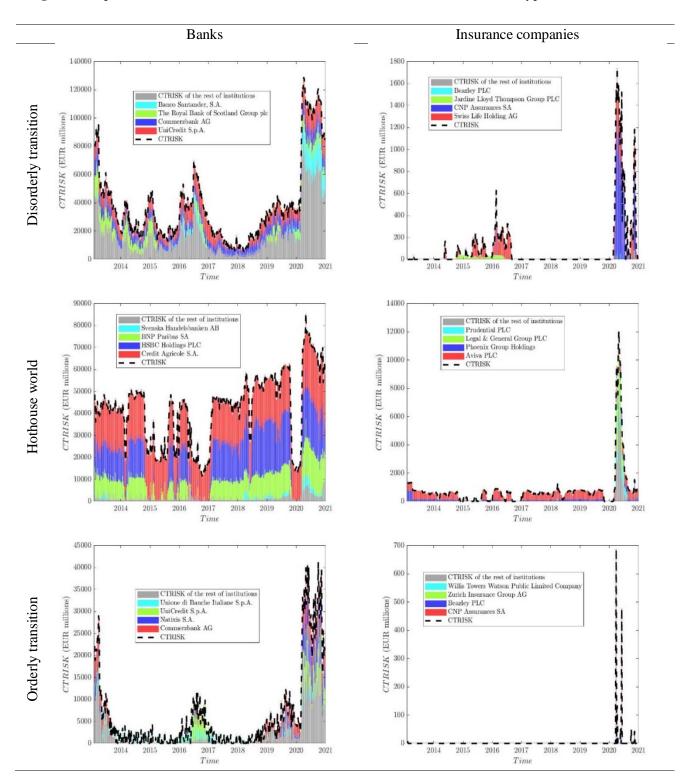
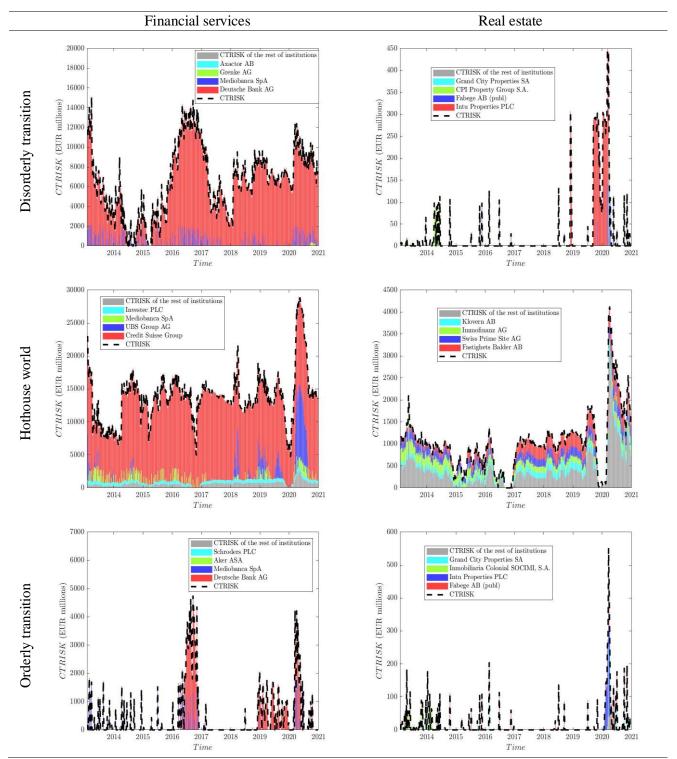


Figure 7. Capital shortfall from climate transition scenarios for financial institution types.

Figure 7. (cont.)



	A. $CTER_i$						
Hothouse world	$\int_{-\infty}^{\infty} r_i \frac{f\left(r_i, r_g \le q_g^{\beta}, r_b \ge q_b^{\alpha}; r_n\right)}{P\left(r_g \le q_g^{\beta}, r_b \ge q_b^{\alpha}; r_n\right)} dr_i$						
Disorderly transition	$\int_{-\infty}^{\infty} r_i \frac{f\left(r_i, r_g \ge q_g^{\beta}, r_b \le q_b^{\alpha}; r_n\right)}{P\left(r_g \ge q_g^{\beta}, r_b \le q_b^{\alpha}; r_n\right)} dr_i$						
Orderly transition	$\int_{-\infty}^{\infty} r_i \frac{f(r_i, q_b^L \le r_b \le q_b^U, q_g^L \le r_g \le q_g^U, q_n^L \le r_n \le q_n^U)}{P(q_b^L \le r_b \le q_b^U, q_g^L \le r_g \le q_g^U, q_n^L \le r_n \le q_n^U, r_i \le CTVaR_i^{\gamma})} dr_i$						
	B. $CTVaR_i^{\gamma}$						
Hothouse world	$F_{i r_g \le q_g^\beta, r_b \ge q_b^\alpha; r_n}^{-1}(\gamma)$						
Disorderly transition	$F_{i r_g \ge q_g^\beta, r_b \le q_b^\alpha; r_n}^{-1}(\gamma)$						
Orderly transition	$F_{i q_{b}^{L}\leq r_{b}\leq q_{b}^{U},q_{g}^{L}\leq r_{g}\leq q_{g}^{U},q_{n}^{L}\leq r_{n}\leq q_{n}^{U}}(\gamma)$						
	C. $CTES_i$						
Hothouse world	$\int_{-\infty}^{CTVaR_{i}^{\gamma}} r_{i} \frac{f\left(r_{i}, r_{g} \leq q_{g}^{\beta}, r_{b} \geq q_{b}^{\alpha}; r_{n}\right)}{P\left(r_{g} \leq q_{g}^{\beta}, r_{b} \geq q_{b}^{\alpha}, r_{i} \leq CTVaR_{i}^{\gamma}; r_{n}\right)} dr_{i}$						
Disorderly transition	$\int_{-\infty}^{CTVaR_{i}^{\gamma}} r_{i} \frac{f\left(r_{i}, r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}; r_{n}\right)}{P\left(r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}, r_{i} \leq CTVaR_{i}^{\gamma}; r_{n}\right)} dr_{i}$						
Orderly transition	$\int_{-\infty}^{CTVaR_i^{\gamma}} r_i \frac{f(r_i, q_b^L \le r_b \le q_b^U, q_g^L \le r_g \le q_g^U, q_n^L \le r_n \le q_n^U)}{P(q_b^L \le r_b \le q_b^U, q_g^L \le r_g \le q_g^U, q_n^L \le r_n \le q_n^U, r_i \le CTVaR_i^{\gamma})} dr_i$						

Table 1. Systemic climate transition risk metrics under different climate transition scenarios.

Notes. The semicolon in the density function $f(\cdot; r_n)$ and the probability of the climate scenario $P(\cdot; r_n)$ indicate that density or probability is defined taking into account possible interactions between the variables that could take place indirectly through the neutral asset (r_n) .

Table 2. Bivariate copula models.

Name	Copula specification	Parameter	Tail dependence
Independent	u_1u_2		_
Gaussian	$\Phi(\Phi^{-1}(u_1), \Phi^{-1}(u_2); \rho)$	ρ	No tail dependence: $\lambda_{\rm U} = \lambda_{\rm L} = 0$
Student t	$T_{\eta}(T_{\eta}^{-1}(u_1), T_{\eta}^{-1}(u_2); \eta, \rho)$	<i>ρ,</i> η	Symmetric tail dependence: $\lambda_L = \lambda_U = 2t_{\eta+1} \left(-\sqrt{(\eta+1)(1-\rho)/(1+\rho)} \right)$
Clayton	$\left(u_1^{- heta}+u_2^{- heta}-1 ight)^{-rac{1}{ heta}}$	θ	$\lambda_L = 2^{-rac{1}{ heta}}, \ \lambda_U = 0$
Gumbel	$\exp\left(-\left(\left(-\log(u_1)\right)^{\theta} + \left(-\log(u_2)\right)^{\theta}\right)^{\frac{1}{\theta}}\right)$	θ	$\lambda_L = 0, \ \lambda_U = 2 - 2^{rac{1}{\Theta}}$
BB1	$\left(1+\left(\left(u_1^{-\theta}-1\right)^{\delta}+\left(u_2^{-\theta}-1\right)^{\delta}\right)^{\frac{1}{\delta}}\right)^{-\frac{1}{\theta}}$	θ,δ	$\lambda_L = 2^{-\frac{1}{ heta\delta}}, \ \lambda_U = 2 - 2^{\frac{1}{\delta}}$

Notes. $\lambda_{U}(\lambda_{L})$ denotes upper (lower) tail dependence. Time-varying dependence is assumed by allowing parameters to change over time, with dynamics given by an ARMA(1,q)-type process (Patton, 2006) for the linear dependence parameter of the Gaussian and student-t copulas, given by $\rho_{t} = \Lambda_{1}\left(\psi_{0} + \psi_{1}\rho_{t-1} + \psi_{2}\frac{1}{q}\sum_{j=1}^{q}\Phi^{-1}(u_{t-j})\Phi^{-1}(v_{t-j})\right)$, where $\Lambda_{1}(x) = \frac{1-\exp(-x)}{1+\exp(-x)}$ is the modified logistic transformation that keeps the value of ρ_{t} in (-1,1), and where $\Phi^{-1}(x)$ is the standard normal quantile function ($\Phi^{-1}(x)$ is replaced by $T_{\eta}^{-1}(x)$ for the student-t copula). For the parameters of the Clayton, Gumbel, and BB1 copulas, we assume that dynamics is given by $\theta_{t} = \Lambda_{2}\left(\overline{\omega}_{\theta} + \overline{\beta}_{\theta}\theta_{t-1} + \overline{\alpha}_{\theta}\frac{1}{q}\sum_{j=1}^{q}|u_{t-j} - v_{t-j}|\right)$ (in the same way for δ in the BB1 copula), where — as in Patton (2006) — q is set to 26 and $\Lambda_{2}(x) = \frac{100}{1+\exp(-x)}$ for the Clayton copula, $\Lambda_{2}(x) = 1 + \frac{99}{1+\exp(-x)}$ for the Clayton, Gumbel, and BB1 copula. We also use 90° rotated copulas for the Clayton, Gumbel, and BB1 to allow for negative dependence. The 90° rotated copula is expressed as $C_{90}(u_{1}, u_{2}) = u_{2} - C(1 - u_{1}, u_{2})$ where $C(\cdot, \cdot)$ is the corresponding standard copula.

	Market assets				Financial firms				
	Green	Neutral	Brown		Banks	Insurance companies	Financial services	Real estate	
Return	0.19%	0.12%	0.01%		-0.07%	0.08%	0.12%	0.08%	
Volatility	2.34%	2.51%	3.09%		5.19%	3.81%	4.17%	4.08%	
Skewness	-1.821	-2.025	-1.544		-0.613	-0.620	-0.773	-0.650	
Kurtosis	16.393	20.259	17.005		11.299	12.789	12.706	18.605	
Max. downturn	-19.27%	-22.27%	-26.00%		-32.46%	-24.49%	-26.86%	-28.86%	
Max. upturn	10.33%	11.02%	12.73%		23.83%	18.56%	19.44%	22.28%	
1 st quartile	-0.83%	-0.90%	-1.45%		-2.69%	-1.69%	-1.86%	-1.69%	
3 rd quartile	1.39%	1.36%	1.59%		2.73%	2.05%	2.31%	2.01%	
10% (left) VaR	-2.81%	-3.09%	-3.94%		-6.72%	-4.80%	-5.22%	-5.14%	
10% (left) ES	-3.92%	-4.28%	-5.41%		-9.18%	-6.60%	-7.20%	-7.07%	
10% (right) VaR	3.19%	3.33%	3.97%		6.59%	4.96%	5.47%	5.30%	
10% (right) ES	4.30%	4.51%	5.43%		9.04%	6.76%	7.45%	7.23%	

Table 3. Summary statistics for returns for different asset classes and financial firms.

Notes. This table presents summary statistics for weekly returns in euros for green, neutral, and brown assets and for European financial firms over the sample period January 2013 to December 2020. For each asset category, we report the average returns, volatility, skewness, kurtosis, maximum downturn and upturn, 10% value-at-risk (VaR), and expected shortfall (ES) for the left and right sides of the return distribution.

		Market assets			Financi	al firms	
_	Green	Neutral	Brown	Banks	Insurance companies	Financial services	Real Estate
Mean							
ϕ_0	0.002*	0.000	-0.001	-0.001	0.001	0.001	0.001
	(0.00)	(0.01)	(0.00)	0.045	0.022	0.054	0.064
ϕ_1		-0.411 (0.92)		0.045	-0.023	-0.054	-0.064
$arphi_1$		(0.92) 0.111* (0.03)		-0.085	0.091	-0.102	-0.093
Volatility dy	namics						
ω	0.000* (0.00)	0.000* (0.07)	0.000* (0.00)	0.000	0.000	0.000	0.000
α_1	0.086* (0.07)	0.012* (0.01)	0.013* (0.09)	0.107	0.085	0.089	0.149
β_1	0.656* (0.32)	0.697* (0.20)	0.795* (0.40)	0.724	0.668	0.664	0.636
δ_1	0.241* (0.08)	0.229* (0.11)	0.190* (0.13)	0.038	0.067	0.052	0.043
Skewed-t dis	tribution						
λ	-0.407* (0.05)	-0.399* (0.05)	-0.315* (0.06)	-0.091	-0.123	-0.104	-0.064
θ	5.692* (1.28)	5.607* (3.23)	7.148* (2.12)	10.067	6.201	5.356	5.739
Goodness-of	-fit						
LogLik	-1078.41	-1047.99	-968.76	-713.51	-816.15	-848.26	-854.06
LJ	[0.67]	[0.98]	[0.71]	[0.65]	[0.61]	[0.60]	[0.62]
LJ2	[0.78]	[0.97]	[0.52]	[0.47]	[0.58]	[0.47]	[0.47]
ARCH-LM K-S	[0.98] [0.84]	[0.99] [0.78]	[0.97] [0.89]	[0.57] [0.89]	[0.67] [0.88]	[0.65] [0.90]	[0.69] [0.88]

Table 4. Maximum likelihood parameter estimates of marginal models for different asset classes and financial firms.

Notes. This table presents parameter estimates of the marginal models for market assets (categorized as green, neutral, and brown) and for European financial firms (banks, insurance companies, financial services, and real estate) as per Eqs. (14)-(15). For markets assets, the z-statistic for the parameter estimates is reported in brackets. Parameter estimates for financial firms are the average of the parameter estimates for each financial firm. For asset markets, an asterisk denotes statistical significance at the 5% level. LogLik, LJ, and LJ2 denote the log-likelihood value of the marginal model, Ljung-Box statistics for serial correlation in the model residuals and in the squared model residuals, respectively, are computed with 20 lags. ARCH effects in the residuals are tested up to the 20th order using Engle's Lagrange multiplier (ARCH-LM) test. KS denotes the Kolmogorov-Smirnov statistic for the null hypothesis of correct model specification (p values in square brackets). For financial institutions, goodness-of-fit information is the average of that information from all marginal models.

	Copula model	Parameter estimates	AIC
$C_{gn}(u_g, u_n)$	BB1	$\hat{\theta} = 1.986^* \ (0.21)$	-794.82
		$\hat{\delta} = 1.885^* \ (0.12)$	
$C_{bn}(u_b,u_n)$	BB1	$\overline{\omega}_{\theta} = 2.554 \ (3.58)$	-741.48
		$\bar{\alpha}_{\theta} = -9.695 \ (18.38)$	
		$\bar{\beta}_{\theta} = 0.294 \ (0.71)$	
		$\overline{\omega}_{\delta} = -2.214^{*} (0.18)$	
		$\bar{\alpha}_{\delta} = 1.029 (2.29)$	
		$\bar{\beta}_{\delta} = 4.232^* \ (0.22)$	
$C_{gb n}(u_g u_n, u_b u_n)$	Independent		0

Table 5. Parameter estimates of bivariate copula models for green, neutral, and brown market assets.

Notes. This table presents parameter estimates of the best copula fit for the copula models in Table 1 for pairings of green, neutral, and brown returns as represented in the upper panel of Figure 2. Standard errors were computed through simulation. An asterisk indicates significance of the parameter at the 1% level. The minimum AIC value adjusted for small-sample bias is reported in the last column.

	Copula model	% institutions	Summary of parameter estimates
$C_{gi n}(u_g u_n,u_i)$	Gaussian	35.8	$\hat{ ho} = 0.13 \ [0.10, 0.18]$
	Student-t	2.6	$\hat{ ho} = 0.16 \ [0.08, 0.24]$
			$\hat{v} = 26.33 [9.95, 50.93]$
	Clayton	33.7	$\hat{\theta} = 0.46 \ [0.24, 0.68]$
	90-Clayton	0.5	$\hat{\theta} = 0.64 \ [0.64, 0.64]$
	Gumbel	4.7	$\hat{\theta} = 1.11 [1.04, 1.18]$
	90-Gumbel	0.5	$\hat{\theta} = 1.16 [1.16, 1.16]$
	Independent	22.2	_
$C_{bi n}(u_b u_n,u_i)$	Gaussian	39.5	$\hat{ ho} = 0.09 \; [0.01, 0.18]$
	Student-t	0.5	$\hat{ ho} = 0.06 \ [0.06, 0.06]$
			$\hat{v} = 8.03 [8.03, 8.03]$
	Clayton	5.3	$\hat{\theta} = 0.33 \; [0.08, 0.61]$
	90-Clayton	8.4	$\hat{ heta} = 0.27 \; [0.04, 0.40]$
	Gumbel	13.2	$\hat{\theta} = 1.16 [1.13, 1.20]$
	90-Gumbel	2.1	$\hat{\theta} = 1.15 [1.09, 1.21]$
	Independent	31.1	
$C_{gb in}(u_g u_i,u_n;u_b u_i,u_n)$	Gaussian	0.5	$\hat{ ho} = -0.02 \; [-0.02, -0.02]$
	Independent	99.5	—

Table 6. Summary of the bivariate copula models for financial firms.

Notes. This table presents a summary of the bivariate copula parameter estimates for the best copula fit between financial firms and the market as represented in the lower (shaded) panel of Figure 2. The third column indicates the percentage of financial firms for which bivariate dependence indicated in the first column is given by the copula function indicated in the second column. The last column reports average copula parameter estimates for the corresponding copula model, with the numbers in square brackets indicating the interquartile range.

	C	Climate transition scenarios						
	Disorderly transition	Hothouse world	Orderly transition					
Panel A. Banks								
CTER	-0.0081 (0.0030)	-0.0081 (0.0155)	-0.0012 (0.0006)					
CTVaR	-0.0722 (0.0189)	-0.0485 (0.0144)	-0.0532 (0.0145)					
CTES	-0.1003 (0.0268)	-0.0792 (0.0232)	-0.0806 (0.0216)					
Panel B. Insurance	ce companies							
CTER	0.0013 (0.0019)	-0.0132 (0.0121)	0.0013 (0.0010)					
CTVaR	-0.0357 (0.0126)	-0.0555 (0.0188)	-0.0371 (0.0120)					
CTES	-0.0609 (0.0210)	-0.0873 (0.0306)	-0.0589 (0.0192)					
Panel C. Financia	al services							
CTER	0.0053 (0.0039)	-0.0265 (0.0130)	0.0012 (0.0016)					
CTVaR	-0.0323 (0.0096)	-0.0664 (0.0169)	-0.0406 (0.0110)					
CTES	-0.0596 (0.0176)	-0.1058 (0.0261)	-0.0645 (0.0173)					
Panel D. Real est	ate							
CTER	0.0221 (0.0103)	-0.0232 (0.0111)	0.0012 (0.0016)					
CTVaR	-0.0225 (0.0089)	-0.0550 (0.0224)	-0.0366 (0.0138)					
CTES	-0.0460 (0.0166)	-0.0851 (0.0334)	-0.0575 (0.0211)					

Table 7. Summary statistics for climate transition systemic risk measures.

Notes. This table presents mean and standard deviation values (in parenthesis) for the three climate transition systemic risk measures, CTER, CTVaR, and CTES, computed weekly over the sample period 2013-2020 for the entire sample and for different categories of financial firms under three different climate transition scenarios.

	Climate transition scenarios									
	Disorderly transition			Hothouse world			Orderly transition			
	CTER	CTVaR	CTES	CTER	CTVaR	CTES	CTER	CTVaR	CTES	
Panel A. Banks										
HSBC	0.0107	-0.0173	-0.0401	-0.0739	-0.1684	-0.2053	0.0001	-0.0288	-0.0388	
BNP Paribas	0.0161	-0.0226	-0.0538	-0.0916	-0.2099	-0.2488	0.0023	-0.0379	-0.0519	
Santander	-0.018	-0.0784	-0.1106	0.0458	-0.033	-0.0637	-0.0038	-0.0589	-0.0902	
Intesa Sanpaolo	-0.0013	-0.0604	-0.0914	0.0132	-0.0604	-0.0934	-0.0029	-0.0611	-0.0918	
Panel B. Insurance										
Alliance	-0.0037	-0.0404	-0.0659	-0.0157	-0.0827	-0.1516	0.0029	-0.0338	-0.0549	
Chubb	0.0071	-0.023	-0.0408	-0.0273	-0.0819	-0.1207	0.0032	-0.0285	-0.0436	
Zurich	0.0001	-0.0343	-0.0571	0.0174	-0.032	-0.057	0.0000	-0.0331	-0.0552	
Axa	-0.0036	-0.0468	-0.072	-0.0026	-0.1122	-0.1696	0.0001	-0.0433	-0.0645	
Panel C. Financial	services									
UBS Group	-0.0059	-0.0492	-0.077	-0.0204	-0.0942	-0.1612	0.0002	-0.0437	-0.0676	
London Stock	0.0297	-0.0111	-0.0399	-0.026	-0.0802	-0.1309	0.0033	-0.0389	-0.0673	
Deutsche Böerse	0.0188	-0.0168	-0.0399	-0.0161	-0.0601	-0.0899	0.002	-0.0353	-0.0566	
Credit Suisse	0.012	-0.0344	-0.0669	-0.087	-0.1939	-0.2429	0.0009	-0.0502	-0.0724	
Panel D. Real estat	te									
Deutsche Wohnen	0.0205	-0.0156	-0.0389	-0.0466	-0.1197	-0.1589	0.0043	-0.0328	-0.0508	
Segro	0.0392	-0.0119	-0.0322	-0.0119	-0.0501	-0.0696	0.0012	-0.0341	-0.051	
Gecina	0.012	-0.0235	-0.0436	-0.0109	-0.05	-0.0722	0.001	-0.0343	-0.0521	
LEG Immobilien	0.0207	-0.0128	-0.0312	-0.0154	-0.0515	-0.0721	0.0027	-0.0299	-0.0455	

Table 8. Average values for climate transition systemic risk for individual institutions.

Notes. This table presents average values for three climate transition systemic risk measures, CTER, CTVaR, and CTES, computed weekly over the 2013-2020 period for the four largest individual firms within each category, considering three different climate transition scenarios.

				Climate tran	sition scenarios			
Disord	n	Hot	Hothouse world			Orderly transition		
	CTRISK	Market Cap.		CTRISK	Market Cap.		CTRISK	Market Cap.
Panel A. Banks								
UniCredit S	8249	27877	Credit Agricole	19274	29675	Commerzbank	1277	10399
Commerzbank	6072	10399	HSBC	13071	142005	Natixis	778	14932
RBS	3519	35177	BNP Paribas	7772	62636	UniCredit S.	745	27877
Santander	2289	70038	Svenska H. AB	525	21385	Unione Banche I	687	4053
Panel B. Insurance								
Swiss Life H. AG	48	8773	Aviva PLC	421	19562	CNP Assurances	3	10818
CNP Assurances	45	10818	Phoenix Group H.	119	3593	Beazley PLC	1	2543
Jardine Lloyd TG	6	3375	Legal General G.	93	17019	Zurich Insurance	0	38941
Beazley PLC	0	2543	Prudential PLC	39	44925	Willis Towers W.	0	13929
Panel C. Financial se	rvices							
Deutsche Bank AG	6141	26307	Credit Suisse	11477	31120	Deutsche Bank AG	334	26307
Mediobanca	332	6719	UBS Group AG	816	52165	Mediobanca	128	6719
Grenke AG	6	2565	Mediobanca	458	6719	Aker ASA	0	2558
Axactor AB	4	182	Investec PLC	445	5513	Schroders PLC	0	9149
Panel D. Real estate								
Intu Properties	18	3452	Fastighets Balder	205	3473	Fabege AB	6	2796
Fabege AB	4	2796	Swiss Prime	155	5164	Intu Properties	4	3452
CPI Property	2	3904	Immofinanz AG	146	2538	I. Colonial	3	2784
Grand City P.	1	2464	Klovern AB	111	1383	Grand City P.	2	2464

Table 9. Average capital shortfall impact of climate transition scenarios on individual firms.

Notes. This table presents mean values (in millions of euros) for capital shortfall as given by the CTRISK for the four most impacted firms in each group under the three climate transition scenarios. Market Cap. denotes average market capitalization over the sample period 2013-2020.

Appendix

A. Proof of Result 1

Proof of Eq. (1). We can express the joint probability $P(r_b \le q_b^{\alpha}, r_g \ge q_g^{\beta}; r_n)$ from integration of the neutral asset as:

$$P\left(r_b \le q_b^{\alpha}, r_g \ge q_g^{\beta}; r_n\right) = \int_{-\infty}^{+\infty} \left(P(r_b \le q_b^{\alpha}|r_n) - P\left(r_b \le q_b^{\alpha}, r_g \le q_g^{\beta}|r_n\right)\right) f(r_n) dr_n$$

where the conditional probabilities can be written using copulas as $P(r_b \le q_b^{\alpha} | r_n) = C_{b|n}(\alpha | u_n)$ and $P\left(r_b \le q_b^{\alpha}, r_g \le q_g^{\beta} | r_n\right) = C_{bg|n}\left(C_{b|n}(\alpha | u_n), C_{g|n}(1 - \beta | u_n)\right)$, where $P\left(r_g \le q_g^{\beta} | r_n\right) = C_{g|n}(1 - \beta | u_n)$ as $P\left(r_g \le q_g^{\beta}\right) = 1 - \beta$. Given that that $u_n = F_n(r_n)$, $du_n = f_n(r_n)dr_n$, it follows that the joint probability in term of copulas is:

$$P(r_{b} \leq q_{b}, r_{g} \geq q_{g}; r_{n}) = \int_{0}^{1} \left\{ C_{b|n}(\alpha|u_{n}) - C_{b,g|n}\left(C_{b|n}(\alpha|u_{n}), C_{g|n}(1-\beta|u_{n})\right) \right\} du_{n}$$

Proof of Eq. (2). We compute $P(q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L, q_n^U \ge r_n \ge q_n^L)$ for a range of quantiles around the median, such that $P(q_b^U \ge r_b \ge q_b^L) = \alpha$, $P(q_g^U \ge r_g \ge q_g^L) = \beta$ and $P(q_n^U \ge r_n \ge q_n^L) = \delta$. Hence, $P(r_b \le q_b^U) = 0.5 + \frac{\alpha}{2}$, $P(r_b \le q_b^L) = 0.5 - \frac{\alpha}{2}$, $P(r_g \le q_g^U) = 0.5 + \frac{\beta}{2}$, $P(r_g \le q_g^L) = 0.5 - \frac{\beta}{2}$.

We can express the joint probability from integration of the neutral asset in the range of quantiles around its median as:

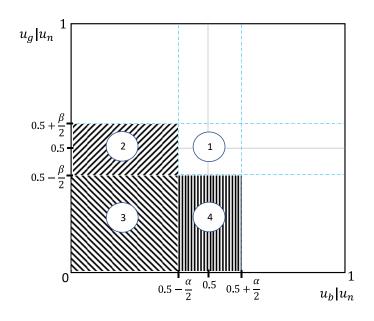
$$P(q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L, q_n^U \ge r_n \ge q_n^L)$$

=
$$\int_{q_n^L}^{q_n^U} P(q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L | r_n) f(r_n) dr_n,$$

where the joint conditional probability $P(q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L | r_n)$ can be decomposed as:

$$\begin{split} P(q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L | r_n) \\ &= P(r_b \le q_b^U, r_g \le q_g^U | r_n) - P(r_b \le q_b^L, r_g \le q_g^L | r_n) \\ &- P(q_b^U \ge r_b \ge q_b^L, r_g \le q_g^L | r_n) - P(r_b \le q_b^L, q_g^U \ge r_g \ge q_g^L | r_n). \end{split}$$

The following figure represents the unit square for the joint distribution between conditional green and brown returns, illustrating the decomposition of the joint probability. The joint conditional probability we are looking for, $P(q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L | r_n)$, is given by box 1, with this box size decomposed as the total size of boxes 1, 2, 3, and 4 ($P(r_b \le q_b^U, r_g \le q_g^U | r_n)$) minus the size of boxes 2 ($P(r_b \le q_b^L, q_g^U \ge r_g \ge q_g^L | r_n)$), 3 ($P(r_b \le q_b^L, r_g \le q_g^L | r_n)$), and 4 ($P(q_b^U \ge r_b \ge q_b^L, r_g \le q_g^L | r_n)$).



Each of those four probabilities can be obtained from conditional copulas as:

a)
$$P(r_b \le q_b^U, r_g \le q_g^U | r_n) = C_{bg|n} \left(C_{b|n} \left(0.5 + \frac{\alpha}{2} | u_n \right), C_{g|n} \left(0.5 + \frac{\beta}{2} | u_n \right) \right)$$

b) $P(r_b \le q_b^L, r_g \le q_g^L | r_n) = C_{bg|n} \left(C_{b|n} \left(0.5 + \frac{\alpha}{2} | u_n \right), C_{g|n} \left(0.5 + \frac{\beta}{2} | u_n \right) \right)$

b)
$$P(r_b \le q_b^L, r_g \le q_g^L | r_n) = C_{bg|n} \left(C_{b|n} \left(0.5 - \frac{\alpha}{2} | u_n \right), C_{g|n} \left(0.5 - \frac{\beta}{2} | u_n \right) \right)$$

c) $P(q^U \ge r_h \ge q^L | r_h \le q^L | r_h) = C_{bg|n} \left(C_{b|n} \left(0.5 - \frac{\alpha}{2} | u_n \right) \right) C_{b|n} \left(0.5 - \frac{\beta}{2} | u_n \right) \right)$

c)
$$P(q_b^U \ge r_b \ge q_b^L, r_g \le q_g^L | r_n) = C_{bg|n} \left(C_{b|n} \left(0.5 + \frac{\alpha}{2} | u_n \right), C_{g|n} \left(0.5 - \frac{\beta}{2} | u_n \right) \right) - C_{bg|n} \left(C_{b|n} \left(0.5 - \frac{\alpha}{2} | u_n \right), C_{g|n} \left(0.5 - \frac{\beta}{2} | u_n \right) \right)$$

d)
$$P(r_b \le q_b^L, q_g^U \ge r_g \ge q_g^L | r_n) = C_{bg|n} \left(C_{b|n} \left(0.5 - \frac{\alpha}{2} | u_n \right), C_{g|n} \left(0.5 + \frac{\beta}{2} | u_n \right) \right) - C_{bg|n} \left(C_{b|n} \left(0.5 - \frac{\alpha}{2} | u_n \right), C_{g|n} \left(0.5 - \frac{\beta}{2} | u_n \right) \right)$$

Hence, the joint conditional probability can be obtained from copulas as:

$$P(q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L | r_n)$$

= $C_{bg|n} (C_{b|n}(a|u_n), C_{g|n}(b|u_n)) + C_{bg|n} (C_{b|n}(d|u_n), C_{g|n}(e|u_n))$
- $C_{bg|n} (C_{b|n}(a|u_n), C_{g|n}(e|u_n)) - C_{bg|n} (C_{b|n}(d|u_n), C_{g|n}(b|u_n))$

where $a = 0.5 + \frac{\alpha}{2}$, $b = 0.5 + \frac{\beta}{2}$, $d = 0.5 - \frac{\alpha}{2}$ and $e = 0.5 - \frac{\beta}{2}$.

Plugging the joint conditional probability into the integral and taking into account that $du_n = f_n(r_n)dr_n$, we can rewrite the joint probability in term of copulas as:

$$\begin{split} P(q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L, q_n^U \ge r_n \ge q_n^L) \\ &= \int_{0.5 - \frac{\delta}{2}}^{0.5 + \frac{\delta}{2}} \Big\{ C_{bg|n} \left(C_{b|n}(a|u_n), C_{g|n}(b|u_n) \right) \\ &+ C_{bg|n} \left(C_{b|n}(d|u_n), C_{g|n}(e|u_n) \right) - C_{bg|n} \left(C_{b|n}(a|u_n), C_{g|n}(e|u_n) \right) \\ &- C_{bg|n} \left(C_{b|n}(d|u_n), C_{g|n}(b|u_n) \right) \Big\} du_n. \end{split}$$

B. Proof of Result 2

Proof of Eq. (3). The joint density between returns for financial firm i and the disorderly transition scenario can be written as:

$$f\left(r_{i}, r_{b} \leq q_{b}^{\alpha}, r_{g} \geq q_{g}^{\beta}; r_{n}\right) = \int_{-\infty}^{\infty} f\left(r_{i}, r_{b} \leq q_{b}^{\alpha}, r_{g} \geq q_{g}^{\beta}|r_{n}\right) f_{n}(r_{n}) dr_{n}$$
$$= \int_{-\infty}^{\infty} f\left(r_{b} \leq q_{b}^{\alpha}, r_{g} \geq q_{g}^{\beta}|r_{i}, r_{n}\right) f(r_{i}|r_{n}) f_{n}(r_{n}) dr_{n}.$$

Note that, consistent with the dependence structure in Figure 1, $f(r_i|r_n) = f_i(r_i)$. Moreover, $f(r_b \le q_b^{\alpha}, r_g \ge q_g^{\beta}|r_i, r_n) = P(r_b \le q_b^{\alpha}|r_i, r_n) - P(r_b \le q_b^{\alpha}, r_g \le q_g^{\beta}|r_i, r_n)$, where last two conditional probabilities can be written in terms of copulas as:

$$P(r_{b} \leq q_{b}^{\alpha} | r_{i}, r_{n}) = C_{b|i,n} (C_{b|n}(\alpha | u_{n}) | u_{i}), \text{ and}$$

$$P(r_{b} \leq q_{b}^{\alpha}, r_{g} \leq q_{g}^{\beta} | r_{i}, r_{n}) = C_{bg|i,n} (C_{b|i,n} (C_{b|n}(\alpha | u_{n}) | u_{i}), C_{g|i,n} (C_{g|n}(1 - \beta | u_{n}) | u_{i})).$$

Since $u_n = F_n(r_n)$, $du_n = f_n(r_n)dr_n$, the joint density can be expressed in terms of copulas as:

$$\begin{split} f\left(r_{i},r_{b} \leq q_{b}^{\alpha},r_{g} \geq q_{g}^{\beta};r_{n}\right) &= \\ &= \int_{0}^{1} \left(C_{b|i,n} (C_{b|n}(\alpha|u_{n})|u_{i}) - C_{bg|i,n} (C_{b|n}(\alpha|u_{n})|u_{i}), C_{g|i,n} (C_{g|n}(1-\beta|u_{n})|u_{i})) \right) f_{i} \left(F_{i}^{-1}(u_{i})\right) du_{n}. \end{split}$$

Proof of Eq. (4). We can express the joint density $f(r_i, q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L, q_n^U \ge r_n \ge q_n^L)$ as:

$$f(q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L, q_n^U \ge r_n \ge q_n^L |r_i) f_i(r_i),$$

where, in turn, the first density of this last expression can be decomposed as:

$$f(q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L, q_n^U \ge r_n \ge q_n^L | r_i)$$
$$= \int_{q_n^L}^{q_n^U} f(q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L | r_n, r_i) f_n(r_n) dr_n.$$

Hence,

$$\begin{split} f\big(r_i, q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L, q_n^U \ge r_n \ge q_n^L\big) \\ &= \int_{q_n^L}^{q_n^U} f\big(q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L | r_n, r_i \big) f_i(r_i) f_n(r_n) dr_n. \end{split}$$

Since $f(q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L | r_n, r_i) = P(q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L | r_n, r_i)$, the joint conditional probability can be expressed in terms of copulas as:

$$\begin{split} P(q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L | r_n, r_i) \\ &= C_{bg|n,i} \left(C_{b|n,i}(a|\{u_n, u_i\}), C_{g|n,i}(b|\{u_n, u_i\}) \right) \\ &+ C_{bg|n,i} \left(C_{b|n,i}(d|\{u_n, u_i\}), C_{g|n,i}(e|\{u_n, u_i\}) \right) \\ &- C_{bg|n,i} \left(C_{b|n,i}(a|\{u_n, u_i\}), C_{g|n,i}(e|\{u_n, u_i\}) \right) \\ &- C_{bg|n,i} \left(C_{b|n,i}(d|\{u_n, u_i\}), C_{g|n,i}(b|\{u_n, u_i\}) \right). \end{split}$$

Using this last expression, and given that $u_n = F_n(r_n)$, $du_n = f_n(r_n)dr_n$, the joint density of the financial firm and the orderly transition scenario can be expressed as:

$$\begin{split} f\left(r_{i}, q_{b}^{U} \geq r_{b} \geq q_{b}^{L}, q_{g}^{U} \geq r_{g} \geq q_{g}^{L}, q_{n}^{U} \geq r_{n} \geq q_{n}^{L}\right) \\ &= \int_{0.5 - \frac{\delta}{2}}^{0.5 + \frac{\delta}{2}} \Big\{ C_{bg|n,i} \left(C_{b|n,i}(a|\{u_{n}, u_{i}\}), C_{g|n,i}(b|\{u_{n}, u_{i}\}) \right) \\ &+ C_{bg|n,i} \left(C_{b|n,i}(d|\{u_{n}, u_{i}\}), C_{g|n,i}(e|\{u_{n}, u_{i}\}) \right) \\ &- C_{bg|n,i} \left(C_{b|n,i}(a|\{u_{n}, u_{i}\}), C_{g|n,i}(e|\{u_{n}, u_{i}\}) \right) \\ &- C_{bg|n,i} \left(C_{b|n,i}(d|\{u_{n}, u_{i}\}), C_{g|n,i}(b|\{u_{n}, u_{i}\}) \right) \Big\} f_{i} \left(F_{i}^{-1}(u_{i}) \right) du_{n}. \end{split}$$

C. Proof of Eq. (5)

$$CTER_{i} = E\left(r_{i} \mid r_{g} \ge q_{g}^{\beta}, r_{b} \le q_{b}^{\alpha}; r_{n}\right) = \int_{-\infty}^{\infty} r_{i} \frac{f\left(r_{i}, r_{g} \ge q_{g}^{\beta}, r_{b} \le q_{b}^{\alpha}; r_{n}\right)}{P\left(r_{g} \ge q_{g}^{\beta}, r_{b} \le q_{b}^{\alpha}; r_{n}\right)} dr_{i}$$
$$= \frac{1}{P\left(r_{g} \ge q_{g}^{\beta}, r_{b} \le q_{b}^{\alpha}; r_{n}\right)} \int_{-\infty}^{\infty} r_{i} \int_{0}^{1} \left[C_{b|i,n}\left(C_{b|n}(\alpha|u_{n})|F_{i}(r_{i})\right)\right) - C_{bg|i,n}\left(C_{b|n}(\alpha|u_{n})|F_{i}(r_{i})\right), C_{g|i,n}\left(C_{g|n}(1-\beta|u_{n})|F_{i}(r_{i})\right)\right)\right] f_{i}(r_{i}) du_{n} dr_{i}.$$

Since $u_i = F_i(r_i)$, $r_i = F_i^{-1}(u_i)$ and $du_i = f_i(r_i)dr_i$, we can write the previous expression as:

$$E\left(r_{i} \mid r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}; r_{n}\right)$$

$$= \frac{1}{\int_{0}^{1} \left\{ C_{b|n}(\alpha|u_{n}) - C_{b,g|n}\left(C_{b|n}(\alpha|u_{n}), C_{g|n}(1-\beta|u_{n})\right) \right\} du_{n}} \int_{0}^{1} F_{i}^{-1}(u_{i}) \int_{0}^{1} \left\{ C_{b|i,n}\left(C_{b|n}(\alpha|u_{n})|u_{i}\right) - C_{bg|i,n}\left(C_{b|n}(\alpha|u_{n})|u_{i}\right), C_{g|i,n}\left(C_{g|n}(1-\beta|u_{n})|u_{i}\right) \right\} du_{n} du_{i}.$$

D. Proof of Eq. (6)

For an orderly climate transition scenario we have:

$$\begin{aligned} CTER_i &= E\left(r_i \mid q_b^L \le r_b \le q_b^U, q_g^L \le r_g \le q_g^U, q_n^L \le r_n \le q_n^U\right) = \\ &= \frac{1}{P\left(q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L, q_n^U \ge r_n \ge q_n^L\right)} \int_{-\infty}^{\infty} r_i f\left(r_i, q_b^U \ge r_b \ge q_b^L, q_g^U \le r_g \ge q_g^L, q_n^U \ge r_n \ge q_n^L\right) dr_i, \end{aligned}$$

where $P(q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L, q_n^U \ge r_n \ge q_n^L)$ is given by Eq. (5). Plugging the value of the joint density $f(r_i, q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L, q_n^U \ge r_n \ge q_n^L)$ as given by Eq. (7) into $CTER_i$, and taking into account that $u_n = F_n(r_n)$, $du_n = f_n(r_n)dr_n$ and $u_i = F_i(r_i)$, $du_i = f_i(r_i)dr_i$, the expected shortfall for an orderly transition can be expressed in terms of copulas as:

$$\begin{split} E(r_i | q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L, q_n^U \ge r_n \ge q_n^L) \\ &= \frac{1}{P(q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L, q_n^U \ge r_n \ge q_n^L)} \int_0^1 \int_{0.5 - \frac{\delta}{2}}^{0.5 + \frac{\delta}{2}} F_i^{-1}(u_i) \\ &\left\{ C_{bg|n,i} \left(C_{b|n,i}(a | \{u_n, u_i\}), C_{g|n,i}(b | \{u_n, u_i\}) \right) \\ &+ C_{bg|n,i} \left(C_{b|n,i}(d | \{u_n, u_i\}), C_{g|n,i}(e | \{u_n, u_i\}) \right) \\ &- C_{bg|n,i} \left(C_{b|n,i}(a | \{u_n, u_i\}), C_{g|n,i}(e | \{u_n, u_i\}) \right) \\ &- C_{bg|n,i} \left(C_{b|n,i}(d | \{u_n, u_i\}), C_{g|n,i}(b | \{u_n, u_i\}) \right) \\ &- C_{bg|n,i} \left(C_{b|n,i}(d | \{u_n, u_i\}), C_{g|n,i}(b | \{u_n, u_i\}) \right) \\ \end{split}$$

E. Proof of Result 3

Proof of Eq. (7). The joint probability $P\left(r_b \leq q_b^{\alpha}, r_g \geq q_g^{\beta}, r_i \leq CTVaR_i^{\gamma}; r_n\right)$ is given by the difference between $P\left(r_b \leq q_b^{\alpha}, r_i \leq CTVaR_i^{\gamma}; r_n\right)$ and $P\left(r_b \leq q_b^{\alpha}, r_g \leq q_g^{\beta}, r_i \leq CTVaR_i^{\gamma}; r_n\right)$. The first probability is defined as:

$$P(r_b \le q_b^{\alpha}, r_i \le CTVaR_i^{\gamma}; r_n) = \int_{-\infty}^{\infty} P(r_b \le q_b^{\alpha}, r_i \le CTVaR_i^{\gamma}|r_n) f_n(r_n) dr_n$$
$$= \int_0^1 C_{bi|n} \left(C_{b|n}(\alpha|u_n), F_i(CTVaR_i^{\gamma}) \right) du_n$$
$$= \int_0^{F_i(CTVaR_i^{\gamma})} \int_0^1 C_{b|i,n} \left(C_{b|n}(\alpha|u_n)|u_i \right) du_n du_i,$$

where, in the second equality, $F_i(CTVaR_i^{\gamma}) = P(r_i \leq CTVaR_i^{\gamma})$. Note that $F_i(CTVaR_i^{\gamma})$ is different from γ as the unconditional distribution of *i* differs from the distribution of *i* conditional on a climate transition scenario, *i. e. CTVaR_i^{\gamma}* is a quantile of that conditional distribution. The second probability can be obtained as:

$$P\left(r_{b} \leq q_{b}^{\alpha}, r_{g} \leq q_{g}^{\beta}, r_{i} \leq CTVaR_{i}^{\gamma}; r_{n}\right)$$

$$= \int_{-\infty}^{CTVaR_{i}^{\gamma}} \int_{-\infty}^{\infty} P\left(r_{b} \leq q_{b}^{\alpha}, r_{g} \leq q_{g}^{\beta} \middle| r_{n}, r_{i}\right) f(r_{i} \mid r_{n}) f_{n}(r_{n}) dr_{n} dr_{i}$$

$$= \int_{0}^{F_{i}(CTVaR_{i}^{\gamma})} \int_{0}^{1} C_{bg\mid i,n} \left(C_{b\mid i,n} \left(C_{b\mid n}(\alpha \mid u_{n}) \middle| u_{i}\right), C_{g\mid i,n} \left(C_{g\mid n}(1 - \beta \mid u_{n}) \middle| u_{i}\right)\right) du_{n} du_{i},$$

where $f(r_i | r_n) = f_i(r_i)$. From the copula representation of those two probabilities, we therefore have:

$$P\left(r_{b} \leq q_{b}^{\alpha}, r_{g} \leq q_{g}^{\beta}, r_{i} \leq CTVaR_{i}^{\gamma}; r_{n}\right)$$

$$= \int_{0}^{F_{i}(CTVaR_{i}^{\gamma})} \int_{0}^{1} \{C_{b|i,n}(C_{b|n}(\alpha|u_{n})|u_{i}) - C_{bg|i,n}(C_{b|n,n}(C_{b|n}(\alpha|u_{n})|u_{i}), C_{g|i,n}(C_{g|n}(1-\beta|u_{n})|u_{i}))\} du_{n} du_{i}.$$

Proof of Eq. (8). Using the joint density $f(r_i, q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L, q_n^U \ge r_n \ge q_n^L)$ in Result 2, we can obtain the joint probability $P(r_i \le CTVaR_i^\gamma, q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L, q_n^U \ge r_n \ge q_n^L)$ as:

$$P(r_{i} \leq CTVaR_{i}^{\gamma}, q_{b}^{U} \geq r_{b} \geq q_{b}^{L}, q_{g}^{U} \geq r_{g} \geq q_{g}^{L}, q_{n}^{U} \geq r_{n} \geq q_{n}^{L})$$

$$= \int_{-\infty}^{CTVaR_{i}^{\gamma}} f(r_{i}, q_{b}^{U} \geq r_{b} \geq q_{b}^{L}, q_{g}^{U} \geq r_{g} \geq q_{g}^{L}, q_{n}^{U} \geq r_{n} \geq q_{n}^{L}) dr_{i}.$$

In terms of copulas, this is:

$$\begin{split} P(r_{i} \leq CTVaR_{i}^{\gamma}, q_{b}^{U} \geq r_{b} \geq q_{b}^{L}, q_{g}^{U} \geq r_{g} \geq q_{g}^{L}, q_{n}^{U} \geq r_{n} \geq q_{n}^{L}) \\ &= \int_{0}^{F_{i}(CTVaR_{i}^{\gamma})} \left\{ \int_{0.5 - \frac{\delta}{2}}^{0.5 + \frac{\delta}{2}} \left\{ C_{bg|n,i} \left(C_{b|n,i}(a|\{u_{n}, u_{i}\}), C_{g|n,i}(b|\{u_{n}, u_{i}\}) \right) \right. \\ &+ C_{bg|n,i} \left(C_{b|n,i}(d|\{u_{n}, u_{i}\}), C_{g|n,i}(e|\{u_{n}, u_{i}\}) \right) \\ &- C_{bg|n,i} \left(C_{b|n,i}(a|\{u_{n}, u_{i}\}), C_{g|n,i}(e|\{u_{n}, u_{i}\}) \right) \\ &- C_{bg|n,i} \left(C_{b|n,i}(d|\{u_{n}, u_{i}\}), C_{g|n,i}(b|\{u_{n}, u_{i}\}) \right) \right\} du_{n} \right\} du_{i}. \end{split}$$

F. Proof of Result 4

Using Result 2 and taking into account that $r_i \leq F_i(CTVaR_i^{\gamma})$, it follows that:

$$f\left(r_{i} \leq F_{i}\left(CTVaR_{i}^{\gamma}\right), r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}; r_{n}\right) = \int_{-\infty}^{CTVaR_{i}^{\gamma}} f\left(r_{i}, r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}; r_{n}\right) dr_{i},$$

and that:

$$f(r_i \leq CTVaR_i^{\gamma}, q_b^U \geq r_b \geq q_b^L, q_g^U \geq r_g \geq q_g^L, q_n^U \geq r_n \geq q_n^L)$$

=
$$\int_{-\infty}^{CTVaR_i^{\gamma}} f(r_i, q_b^U \geq r_b \geq q_b^L, q_g^U \geq r_g \geq q_g^L, q_n^U \geq r_n \geq q_n^L) dr_i,$$

where $f(r_i, r_g \ge q_g^\beta, r_b \le q_b^\alpha; r_n)$ and $f(r_i \le CTVaR_i^\gamma, q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L, q_n^U \ge r_n \ge q_n^L)$ are given by Result 2.

G. Proof of Eq. (11)

 $P\left(r_{i} \leq CTVaR_{i}^{\gamma} \middle| r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}; r_{n}\right) \text{ is given by copulas as the ratio between } P\left(r_{i} \leq CTVaR_{i}^{\gamma}, r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}; r_{n}\right) \text{ and the conditioning probability } P\left(r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}; r_{n}\right), \text{ which can be expressed in terms of copulas as shown in the proofs of Results 1 and 3. Thus, } P\left(r_{i} \leq CTVaR_{i}^{\gamma} \middle| r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}; r_{n}\right) \text{ can be written as:}$

$$\frac{\int_{0}^{F_{i}(CTVaR_{i}^{\gamma})}\int_{0}^{1}\left\{C_{b|i,n}(C_{b|n}(\alpha|u_{n})|u_{i})-C_{b,g|i,n}(C_{b|n,n}(C_{b|n}(\alpha|u_{n})|u_{i}),C_{g|i,n}(C_{g|n}(1-\beta|u_{n})|u_{i}))\right\}du_{n}du_{i}}{\int_{0}^{1}\left\{C_{b|n}(\alpha|u_{n})-C_{b,g|n}(C_{b|n}(\alpha|u_{n}),C_{g|n}(1-\beta|u_{n}))\right\}du_{n}}$$

The value of this ratio is a function of $F_i(CTVaR_i^{\gamma})$. We denote the ratio as a function $G\left(F_i(CTVaR_i^{\gamma})\right)$. Since $G\left(F_i(CTVaR_i^{\gamma})\right) = \gamma$, then $F_i(CTVaR_i^{\gamma}) = G^{-1}(\gamma)$. Hence, $CTVaR_i^{\gamma} = F_i^{-1}(G^{-1}(\gamma))$.

H. Proof of Eq. (12)

In a disorderly transition, $CTES_i^{\gamma}$ is given by:

$$\begin{split} E\left(r_{i}\middle|r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}, r_{i} \leq CTVaR_{i}^{\gamma}; r_{n}\right) = \\ = \frac{1}{P\left(r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}, r_{i} \leq CTVaR_{i}^{\gamma}; r_{n}\right)} \int_{-\infty}^{CTVaR_{i}^{\gamma}} r_{i} f\left(r_{i}, r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}; r_{n}\right) dr_{i}. \end{split}$$

We can rewrite the joint density in the previous expression as:

$$f\left(r_{i}, r_{g} \ge q_{g}^{\beta}, r_{b} \le q_{b}^{\alpha}; r_{n}\right) = \int_{-\infty}^{\infty} f\left(r_{i}, r_{b} \le q_{b}^{\alpha}, r_{g} \ge q_{g}^{\beta} | r_{n}\right) f_{n}(r_{n}) dr_{n}$$
$$= \int_{-\infty}^{\infty} f\left(r_{b} \le q_{b}^{\alpha}, r_{g} \ge q_{g}^{\beta} | r_{i}, r_{n}\right) f(r_{i} | r_{n}) f_{n}(r_{n}) dr_{n},$$

where $f(r_i|r_n) = f_i(r_i)$ and $f(r_b \le q_b, r_g \ge q_g|r_i, r_n) = P(r_b \le q_b^{\alpha}|r_i, r_n) - P(r_b \le q_b^{\alpha}, r_g \le q_g^{\beta}|r_i, r_n)$. Those last two conditional probabilities can be written in terms of copulas as:

$$P(r_{b} \leq q_{b}^{\alpha}|r_{i}, r_{n}) = C_{b|i,n} (C_{b|n}(\alpha|u_{n})|u_{i}),$$

$$P(r_{b} \leq q_{b}^{\alpha}, r_{g} \leq q_{g}^{\beta}|r_{i}, r_{n}) = C_{bg|i,n} (C_{b|i,n} (C_{b|n}(\alpha|u_{n})|u_{i}), C_{g|i,n} (C_{g|n}(1-\beta|u_{n})|u_{i})).$$

Now, plugging those results into $\int_{-\infty}^{\infty} r_i f(r_i, r_g \ge q_g^{\beta}, r_b \le q_b^{\alpha}, r_i; r_n) dr_i$, and taking into account that $u_i = F_i(r_i), du_i = f_i(r_i) dr_i$, we can write

$$\int_{-\infty}^{CTVaR_{i}^{\gamma}} r_{i} f\left(r_{i}, r_{g} \geq q_{g}^{\beta}, r_{b} \leq q_{b}^{\alpha}, r_{i}; r_{n}\right) dr_{i}$$

$$= \int_{0}^{F_{i}(CTVaR_{i}^{\gamma})} \int_{0}^{1} F_{i}^{-1}(u_{i}) \left\{ C_{b|i,n}(C_{b|n}(\alpha|u_{n})|u_{i}) - C_{bg|i,n}(C_{b|i,n}(C_{b|n}(\alpha|u_{n})|u_{i}), C_{g|i,n}(C_{g|n}(1-\beta|u_{n})|u_{i})) \right\} du_{n} du_{i}.$$

I. Proof of Eq. (13)

$$\begin{split} & E\left(r_{i}\middle|r_{i} \leq CTVaR_{i}^{\gamma}, q_{b}^{U} \geq r_{b} \geq q_{b}^{L}, q_{g}^{U} \geq r_{g} \geq q_{g}^{L}, q_{n}^{U} \geq r_{n} \geq q_{n}^{L}\right) \\ & = \frac{1}{P\left(r_{i} \leq CTVaR_{i}^{\gamma}, q_{b}^{U} \geq r_{b} \geq q_{b}^{L}, q_{g}^{U} \geq r_{g} \geq q_{g}^{L}, q_{n}^{U} \geq r_{n} \geq q_{n}^{L}\right)} \int_{-\infty}^{CTVaR_{i}^{\gamma}} r_{i} f\left(r_{i}, q_{b}^{U} \geq r_{b} \geq q_{b}^{L}, q_{g}^{U} \geq r_{g} \geq q_{g}^{L}, q_{n}^{U} \geq r_{n} \geq q_{n}^{L}\right) dr_{i}. \end{split}$$

According to Result 2, we can rewrite the joint density in the previous expression as:

$$f(r_i, q_b^U \ge r_b \ge q_b^L, q_g^U \ge r_g \ge q_g^L, q_n^U \ge r_n \ge q_n^L)$$

= $\int_{0.5 - \frac{\delta}{2}}^{0.5 + \frac{\delta}{2}} C(d \le u_b \le a, e \le u_g \le b | \{u_n, u_i\}) f_i(F_i^{-1}(u_i)) du_n$

Now, taking into account that $u_i = F_i(r_i)$, $du_i = f_i(r_i)dr_i$, we have:

$$\begin{split} & E(r_{i}|r_{i} \leq CTVaR_{i}^{\gamma}, q_{b}^{U} \geq r_{b} \geq q_{b}^{L}, q_{g}^{U} \geq r_{g} \geq q_{g}^{L}, q_{n}^{U} \geq r_{n} \geq q_{n}^{L}) \\ & = \frac{1}{P(r_{i} \leq CTVaR_{i}^{\gamma}, q_{b}^{U} \geq r_{b} \geq q_{b}^{L}, q_{g}^{U} \geq r_{g} \geq q_{g}^{L}, q_{n}^{U} \geq r_{n} \geq q_{n}^{L})} \int_{0}^{F_{i}(CTVaR_{i}^{\gamma})} \int_{0.5-\frac{\delta}{2}}^{0.5+\frac{\delta}{2}} F_{i}^{-1}(u_{i})C(d_{i}) \\ & \leq u_{b} \leq a, e \leq u_{g} \leq b|\{u_{n}, u_{i}\})du_{n} du_{i}. \end{split}$$