## Can investors curb greenwashing?

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> GRASFI 2024 Annual Conference Singapore, September 2–4, 2024

# Greenwashing: a major issue

**Greenwashing**: The practice by which companies claim they are doing more for the environment than they actually are. (European Commission).



→ Annual screening of company websites (European Commission, 2021): In 42% of cases, the authorities "had reason to believe that the [company's] claim may be false or deceptive."

## Why greenwashing?

- At equilibrium, environmentally well-rated companies benefit from lower costs of capital (Pástor et al., 2021; Pedersen et al., 2021; Zerbib, 2022).
- 2. The reliability of environmental scores is questionable (Berg et al., 2022):
  - companies' environmental footprints are challenging to measure accurately,
  - measurement methods are not standardized.
- Companies can benefit from information asymmetry about their true environmental values (Barbalau and Zeni, 2023) and communicate in an ambiguous manner (Fabrizio and Kim, 2019).
- Companies have the ability and the incentive to overstate their environmental value.

# Greenwashing: a major issue

#### For investors: major obstacle to

- (i) environment-related risk assessment;
- (ii) environmental impact of investments.

#### Questions:

- What are the incentives for companies to greenwash?
- When do companies use environmental communication to greenwash?
- What role can investors play in influencing greenwashing practices?

### What we do

- We build a dynamic asset pricing equilibrium model with
  - Information asymmetry about companies' environmental value;
  - Companies which can (i) communicate and (ii) reduce their emissions to influence their environmental score;
  - ► A representative investor (i) with pro-environmental preferences and (ii) who can penalize revealed environmental misrating (through the occurrence of controversies).

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- 2. We extend the model allowing for **interaction between companies** and solve the resulting game.
- 3. **We validate empirically** the environmental communication dynamics of green companies.

### Contributions to the literature

- Greenwashing and environmental disclosure: Duflo et al. (2013); Duchin et al. (2023); Hoepner et al. (2017); Bingler et al. (2022, 2023) and Flammer (2021); Ilhan et al. (2023); Berg et al. (2022, 2021).
  - First theoretical paper linking greenwashing to investment decisions with Chen (2023).
- Sustainable asset pricing: Pástor et al. (2021); Pedersen et al. (2021); Zerbib (2022); Bolton and Kacperczyk (2021); De Angelis et al. (2023); Pástor et al. (2022); Zerbib (2022); Cheng et al. (2023); Avramov et al. (2022); Sauzet and Zerbib (2022); Berk and van Binsbergen (2021); Goldstein et al. (2022); Pástor et al. (2022); Ardia et al. (2023); Van der Beck (2023).
  - Correction for greenwashing in addition to green premium on expected returns.
- Asset pricing and information asymmetry: Grossman and Stiglitz (1980); Admati and Pfleiderer (1986); Hughes (1986); Easley and O'hara (2004); Lambert et al. (2012).
  - Asset pricing model with random revelation times.
- Impact investing: De Angelis et al. (2023); Hartzmark and Shue (2023); Favilukis et al. (2023); Green and Roth (2024); Oehmke and Opp (2024); Green and Roth (2024); Landier and Lovo (2023); Edmans et al. (2023); Barber et al. (2021); Bonnefon et al. (2022); Heeb et al. (2023).
  - Double positive impact of investors: curb greenwashing & foster abatement.

### **Outline**

1 A dynamic equilibrium model with corporate greenwashing

2 Optimal greenwashing and investor's impact

3 Empirical evidence

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1 A dynamic equilibrium model with corporate greenwashing

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## Market setting

Probability space  $(\Omega, \mathbb{F} = (\mathcal{F}_t)_{t>0}, \mathbb{P})$  with **infinite** time horizon.

#### Assets:

- 1 risk-free asset with zero interest rate
- n firms issuing stocks at quantity normalized to 1, indexed by i

**Price process** of the risky assets,  $S \in \mathbb{R}^n$ :

$$dS_t = \mu_t dt + \sigma dB_t,$$

- $\mu_t \in \mathbb{R}^n$  vector of expected returns, determined at equilibrium
- $\sigma \in \mathbb{R}^{n \times n}$  exogenously specified constant volatility matrix
- $B \in \mathbb{R}^n$  a.s. a Brownian motion

### Environmental score

<u>Fundamental environmental value</u> of company *i*:

$$dV_t^i = \underbrace{v_t^i dt}_{\text{Abatement effect}}, \quad V_0^i = p^i,$$

with  $v^i$  the emissions reduction (or abatement) effort of company i.

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**BUT information asymmetry:** the environmental value is UNKNOWN by the investor. **Proxy** for this value:

Environmental score of company *i*:  $E_0^i = q^i$ ,

$$dE_t^i = \underbrace{a(V_t^i - E_t^i)dt}_{\text{Rating agency effect}} + \underbrace{(V_{t-}^i - E_{t-}^i)\Theta_t^i dN_t^i}_{\text{Controversy effect}} + \underbrace{c_t^i dt}_{\text{Communication effect}} + \underbrace{zdW_t^i}_{\text{Measurement error}}$$

- c<sup>i</sup> the **environmental communication effort** of company i
- N<sup>i</sup> Poisson process, W<sup>i</sup> Brownian motion, independent from each other
- $\Theta_t^i \in [0, 1]$  random fraction of misrating revealed at controversy,  $\mathbb{E}[\Theta_t^i] := b$ .

## Misrating proxy

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 $\Rightarrow$  use of **controversies history** which reveal a random fraction of the ongoing misrating (through jumps of  $N^i$ ).

#### Misrating proxy:

$$dM_t^i = \underbrace{-\rho M_t^i dt}_{\text{Forgetting rate}} + \underbrace{(E_t^i - E_{t-}^i)^2 dN_t^i}_{\text{Square of misrating revealed by controversies}}, \qquad M_0^i = u^i$$

# Formal definition of greenwashing

### Greenwashing

Company *i* is *greenwashing* at time *t* if:

- (i) it is not underrated, that is,  $E_t^i \geq V_t^i$ ,
- (ii) its environmental communication is positive,  $c_t^i > 0$ ,
- (iii) it communicates more than it abates,  $c_t^i > v_t^i$ .

When the company is greenwashing, its *greenwashing effort* is defined as  $c_t^i - v_t^i$ .

⇒ Greenwashing is any communication effort that aims at creating or increasing a positive gap between the environmental score and the fundamental environmental value, when the company is accurately rated or already overrated.

# Investor's program

*Notations*: all variables are  $\in \mathbb{R}^n$  in this slide.

$$\sup_{\omega \in \mathbb{A}^{\omega}} \mathbb{E} \left[ \int_{0}^{\infty} e^{-rt} \left\{ \underbrace{\omega_{t}' dS_{t} - \frac{\gamma}{2} \langle \omega' dS \rangle_{t}}_{\text{Mean-variance criterion}} + \underbrace{\omega_{t}' (\beta E_{t} - \alpha M_{t}) dt}_{\text{Non-pecuniary preferences}} \right\} \right]$$

Mean-variance criterion (Standard, e.g., Bouchard et al., 2018)

### Non-pecuniary preferences:

- Pro-environmental preferences, βE<sub>t</sub> (e.g., Pástor et al., 2021; Zerbib, 2022)
- Penalty on revealed misrating,  $-\alpha M_t$
- $\Rightarrow$  Expected returns  $\mu_t \in \mathbb{R}^n$  determined at equilibrium

# Company i's program

*Notations*: the exponent *i* indicates the *i*-th component of a vector.

**Objective**: Trade-off between reducing its **cost of capital**  $\mu^{i}$  and the **quadratic costs** of environmental efforts

$$\inf_{(r^i,c^i)\in\mathbb{A}}\mathbb{E}\left[\int_0^\infty e^{-\delta t}\left(\mu^i_t+\frac{\kappa^i_v}{2}(v^i_t)^2+\frac{\kappa^i_c}{2}(c^i_t)^2\right)dt\right],$$

- $\mu_t^i$ : expected returns of company *i* determined at equilibrium
- $\frac{\kappa_t^i}{2}(v_t^i)^2$ : quadratic costs of abatement effort,  $v_t^i$
- $\frac{\kappa_c^i}{2}(c_t^i)^2$ : quadratic costs of communication effort,  $c_t^i$

Equivalent program with asset prices

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# Optimal portfolio and equilibrium expected returns

### Proposition

The optimal asset allocation of the investor is the pointwise solution

$$\omega_t^* = \frac{1}{\gamma} \Sigma^{-1} (\mu_t + \beta E_t - \alpha M_t),$$

and the equilibrium expected return is

$$\mu_t = \gamma \Sigma \mathbf{1}_n - \boldsymbol{\beta} E_t + \boldsymbol{\alpha} M_t.$$

 $\beta E_t$ : Green premium on expected returns (Pástor et al., 2021; Zerbib, 2022).

 $\alpha M_t$ : Additional correction for greenwashing companies.



# Companies' program with explicit objective

Knowing equilibrium expected returns, companies' program becomes:

$$\inf_{(\boldsymbol{r}^i,\boldsymbol{c}^i)\in\mathbb{A}}\mathbb{E}\left[\int_0^\infty e^{-\delta t}\left(\gamma \boldsymbol{\Sigma} \mathbf{1}_n - \boldsymbol{\beta}\boldsymbol{E}_t^i + \boldsymbol{\alpha}\boldsymbol{M}_t^i + \frac{\kappa_r^i}{2}(\boldsymbol{r}_t^i)^2 + \frac{\kappa_c^i}{2}(\boldsymbol{c}_t^i)^2\right)dt\right].$$

Under the following constraints:

$$\begin{cases} dE_t^i = \textit{a}(V_t^i - E_t^i) \textit{d}t + (V_{t-}^i - E_{t-}^i) \Theta_t^i \textit{d}N_t^i + c_t^i \textit{d}t + \textit{z} \textit{d}W_t^i, & E_0^i = \textit{q}^i, \\ dV_t^i = v_t^i \textit{d}t, & V_0^i = \textit{p}^i, \\ dM_t^i = -\rho M_t^i \textit{d}t + (E_t^i - E_{t-}^i)^2 \textit{d}N_t^i, & M_0^i = \textit{u}^i, \\ \mathbb{A} := \left\{ (\textit{c}, \textit{v}) \in \mathbb{R}^2, \mathbb{F} - \text{prog. meas.} : \mathbb{E}[\int_0^\infty e^{-\delta^i \wedge \delta t} \left( |\textit{c}_t|^2 + |\textit{v}_t|^2 \right) \textit{d}t] < \infty \right\} \end{cases}$$

 $\Rightarrow$  Each company looks for  $r^i$  and  $c^i$  that maximize its environmental score,  $E^i$ , controlling for its misrating proxy,  $M^{i}$ , and costs of environmental action (abatement and communication),  $\frac{\kappa_r^i}{2} (\mathbf{v}_t^i)^2 + \frac{\kappa_c^i}{2} (\mathbf{c}_t^i)^2$ .

## Optimal strategies

### Proposition (Optimal strategies)

The optimal environmental communication effort,  $c^{i,*}$ , and abatement effort,  $v^{i,*}$ , of company i are as follows:

$$\begin{split} \boldsymbol{c}_t^{i,*} &= \frac{1}{\kappa_c^i} \left( \boldsymbol{B}^i - \boldsymbol{A}^i (\boldsymbol{E}_t^{i,*} - \boldsymbol{V}_t^{i,*}) \right), \\ \boldsymbol{v}_t^{i,*} &= \frac{1}{\kappa_v^i} \left( \frac{\beta}{\delta} - \boldsymbol{B}^i + \boldsymbol{A}^i (\boldsymbol{E}_t^{i,*} - \boldsymbol{V}_t^{i,*}) \right), \end{split}$$

where

$$B^{i} = \frac{\beta(1 + \frac{A^{i}}{\delta \kappa_{V}^{i}})}{\delta + a + b\lambda^{i} + \frac{2A^{i}}{\tilde{\kappa}^{I}}}, \qquad A^{i} = \frac{\tilde{\kappa}^{i}}{4} R^{i} \left(\sqrt{1 + \frac{16}{\tilde{\kappa}^{i}}} \frac{T^{i}}{(R^{i})^{2}} - 1\right)$$
$$T^{i} = \frac{2\lambda^{i} b^{2} \alpha}{(1 + b)(\delta + \rho)}, \quad R^{i} = \delta + 2a + \frac{2\lambda^{i} b}{1 + b}, \quad \tilde{\kappa}^{i} = \frac{2}{\frac{1}{\kappa^{I}} + \frac{1}{\kappa^{I}}}$$

with  $E^{i,*}, V^{i,*}$  state variables when the optimal strategies  $c^{i,*}, v^{i,*}$  are employed,  $A^i, B^i \geq 0$  and  $\frac{\beta}{\delta} - B^i \geq 0$ .



# Optimal greenwashing effort when $\beta > 0$ , $\alpha > 0$

## Proposition (Greenwashing effort)

If the following condition (\*) is satisfied,

$$\frac{\kappa_V^i}{\kappa_C^i} > \frac{a + b\lambda^i}{\delta},\tag{*}$$

company i greenwashes if, and only if,

$$0 \leq E_t^{i,*} - V_t^{i,*} < \frac{1}{\frac{2}{\bar{\kappa}^j} A^i} G_{max}^i, \qquad G_{max}^i = \frac{2}{\bar{\kappa}^i} B^i - \frac{\beta}{\delta \kappa_v^i}.$$

When it greenwashes, its greenwashing effort is as follows:

$$c_t^{i,*} - v_t^{i,*} = G_{max}^i - \frac{2}{\bar{\kappa}^i} A^i (E_t^{i,*} - V_t^{i,*})$$

When condition (\*) is not satisfied, company i never greenwashes.

NB:  $a + b\lambda^i \equiv$  Revelation intensity (inverse: degree of information asymmetry).

⇒ Companies greenwash to maintain their environmental score at a certain level above their environmental value

## Impact of investor's preferences and penalty

- β Sensitivity of pro-environmental preferences of the investor
- α Investor's penalty on revealed misrating

## Proposition (Investor's impact on greenwashing)

When condition (\*) is satisfied, the maximal greenwashing effort,  $G_{max}^i$ , increases linearly in  $\beta$  and decreases in a convex way in  $\alpha$ .

## Proposition (Investor's impact on abatement)

The constant part in the abatement effort,  $\frac{1}{\kappa'_i}\left(\frac{\beta}{\delta}-B^i\right)$ , increases linearly in  $\beta$ , and, when condition (\*) is satisfied, increases in a concave way in  $\alpha$ .

 $\Rightarrow$  Adds to the impact investing literature (Landier and Lovo, 2023; Green and Roth, 2024; Pástor et al., 2022; De Angelis et al., 2023; Oehmke and Opp, 2024).

# Impact of investors on greenwashing and abatement

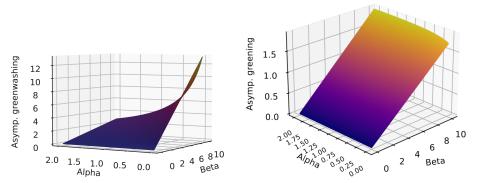


Figure: Average greenwashing and abatement as a function of  $\beta$  and  $\alpha$ . Asymptotic expected optimal greenwashing ( $\lim_{t\to\infty}\mathbb{E}[c_t^*-v_t^*]$ ; left) and abatement ( $\lim_{t\to\infty}\mathbb{E}[v_t^*]$ ; right) as a function of the pro-environmental sensitivity,  $\beta$ , and the misrating penalty,  $\alpha$ .

- Greenwashing and abatement efforts increase linearly with green preferences  $\beta$ .
  - ullet Penalty  ${\color{blue} \alpha}$  strongly deters greenwashing, and encourages abatement.
- Calibration , which verifies condition (\*), and  $\kappa_V/\kappa_c=50$ .

# Greenwashing and transparency parameters

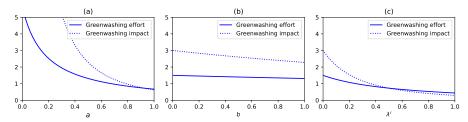


Figure: **Greenwashing and transparency parameters when**  $\alpha = 0$ . The maximum greenwashing effort,  $G^i_{max}$ , (solid lines), and greenwashing impact,  $\lim_{t \to \infty} \mathbb{E}[E^{i,*}_t - V^i_t,*]$ , (dotted lines), as a function of transparency parameters  $a, b, \lambda^i$ , when the investor's penalty,  $\alpha$ , is null.

- 1. Without investor's penalty on misrating ( $\alpha = 0$ ):
  - The rating agency's efficiency, a, strongly deters greenwashing effort & impact.
  - Controversy frequency,  $\lambda^i$ , and portion of misrating revealed,  $\boldsymbol{b}$ , are also dissuasive but with a smaller magnitude.

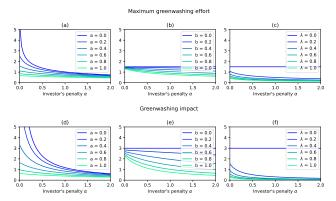


Figure: Greenwashing and penalty  $\alpha$  for various transparency parameters. The maximum greenwashing effort,  $G^i_{max}$ , and greenwashing impact,  $\lim_{t\to\infty} \mathbb{E}[E^i_t, -V^i_t]$ , as a function of the investor's penalty,  $\alpha$ , for different values of transparency parameters  $a, b, \lambda^i$ .

#### 2. With investor's penalty on misrating ( $\alpha > 0$ ):

- The effect of **a** replaces rather than cumulates with the penalty  $\alpha$ .
- The existence of controversies ( $\lambda^i > 0$ , b > 0) is *necessary* for  $\alpha$  to have an impact, and the magnitude of  $\lambda^i$ , b amplify the impact of the penalty  $\alpha$ .

## Greenwashing and technological change

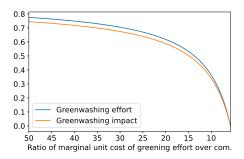


Figure: **Greenwashing and technological change.** Maximum greenwashing effort,  $G^i_{max}$ , and impact,  $\lim_{t\to\infty}\mathbb{E}[E^{i,*}_t-V^{i,*}_t]$ , as function of the ratio of marginal unit costs of abatement and communication  $\kappa^i_r/\kappa^i_c$ . Consistently with Proposition 3.3, greenwashing is zero when the threshold represented by condition (\*) is hit.

 $\Rightarrow$  Curbing greenwashing through green technological change would require a sustained and pronounced R&D effort to bring down  $\kappa_r^i$  before being effective on greenwashing effort and impact.

### What if environmental scores were normalized?

#### Extension of the investor's program:

$$\sup_{\omega \in \mathbb{A}^{\omega}} \mathbb{E} \left[ \int_{0}^{\infty} e^{-rt} \left\{ \underbrace{\omega_{t}' dS_{t} - \frac{\gamma}{2} \langle \omega' dS \rangle_{t}}_{\text{Mean-variance criterion}} + \underbrace{\omega_{t}' \left(\beta \frac{E_{t}}{h(\frac{1}{n} \sum_{i} E_{t}^{i})} - \alpha M_{t}\right) dt}_{\text{Non-pecuniary preferences}} \right\} \right],$$

*h* a regular function approximating identity on  $\mathbb{R}_+$ .

#### Two interpretations:

- Investors practice a "best-in-class" strategy.
- Rating agencies standardize environmental scores.

### Method and results

#### **Resolution approach** of the *n*-player game:

- approximate with the mean field limit  $(n \to \infty)$ ,
- show that there exists a unique Nash equilibrium in the equivalent mean field game.

#### Main results:

- Qualitatively, optimal abatement, communication and greenwashing efforts follow the same pattern as in the baseline case
- 2. However, **all efforts are lower** at the Nash equilibrium, as cross-sectional comparison decreases the incentive to get high environmental ratings.

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## Empirical analysis

**Challenge**: No robust, exhaustive, and dynamic data on companies' emission abatement. ⇒ Unreliable test for greenwashing

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**Challenge**: No robust, exhaustive, and dynamic data on companies' emission abatement.  $\Rightarrow$  Unreliable test for greenwashing

However, we build a proxy for environmental communication effort,  $\hat{c}_t^i$ , and:

- analyze its strength;
- 2. test the dynamics of the model:

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#### Monthly data from Covalence:

- an environmental reputation score, Rep ∈ [0, 100];
- an environmental controversy score, Con ∈ [0, 100];
- an environmental performance score, E ∈ [0, 100].

**Sample**: 3,769 global companies between December 2015 and December 2022: 145,508 firm×month observations.

#### **Empirical Method**

#### We build a two-step method:

- Step 1: Build a proxy for the environmental communication effort, out of Rep and Con
  - $\Rightarrow$  Analyze  $\hat{c}_t^i$

- Step 2: Test the dynamics of environmental communication effort
  - $\Rightarrow$  Test the equilibrium equation based on  $\hat{c}_t^i$

#### Method: Step 2 (Dynamics of env. comm. effort)

Recall, we want to test:

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To address simultaneity issues, we estimate:

$$\Delta \hat{\mathbf{c}}_t^i = \alpha_3^i + \iota_{3,t} + \beta_3 \Delta \mathbf{E}_t^{i,*} + \varepsilon_{3,t}^i,$$

where  $\Delta E_t^{i,*}$  is the prediction of the following regression:

$$\Delta E_t^i = \alpha_4^i + \beta_4 E_{t-2}^i + \varepsilon_{4,t}^i.$$

### Summary of results from the empirics

#### **Conclusions** about environmental communication:

- Companies have implemented a quasi-structural positive envir. com. policy
- 2. Counter-cyclical dynamic of the envir. com., as highlighted by the model
- ⇒ Supported by the low marginal unit cost of communication and the asymmetry of information (Barbalau and Zeni, 2023), the **greenwashing** option, at least part of the time, is the most likely.

#### Conclusion

- Investors' pro-environmental preferences incentivize companies to greenwash
  - Impeding further abatement efforts
- Investors can curb greenwashing practices by penalizing misrating revealed by controversies
  - ► This, in turn, encourages abatement
- · Policymakers can also curb greenwashing and increase abatement:
  - (i) regulations strengthening transparency
  - (ii) support for environmental technological innovation
- These results are qualitatively robust to the introduction of an interaction between companies; however, standardization of environmental ratings seems detrimental to abatement efforts.
- Empirical results suggest that companies tend to greenwash significantly.

# Thank you!

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## Companies' program in terms of asset prices

Company i's program is equivalent to the following:

$$\sup_{(r^i,c^i)\in\mathbb{A}}\mathbb{E}\left[\int_0^\infty e^{-\delta t}\left(\delta(S_0^i-S_t^i)-\frac{\kappa_r^i}{2}(r_t^i)^2-\frac{\kappa_c^i}{2}(c_t^i)^2\right)dt\right],$$

with  $S_0^i$  the initial price of the asset issued by company *i*.



# Equilibrium expected returns: Sketch of the proof

#### Definition (Equilibrium expected returns)

#### $\mu$ so that:

- the investor implements her optimal investing strategy  $\omega^*$ ,
- market clears:  $\forall i, \ \forall t, \ \omega_t^{*,i} = 1$ .

#### Proof.

- Define the candidate optimal strategy  $\omega_t^* := \frac{1}{\gamma} \Sigma^{-1} (\mu_t + \beta E_t \alpha M_t)$ .
- The investor's program can be rewritten as

$$\sup_{\omega \in \mathbb{A}^{\omega}} \mathbb{E} \left[ \int_0^{\infty} e^{-\delta' t} \left\{ -\frac{\gamma}{2} (\omega_t - \omega_t^*)' \Sigma(\omega_t - \omega_t^*) + \frac{\gamma}{2} \omega_t^{*'} \Sigma \omega_t^* \right\} dt \right].$$

- $\Rightarrow$  The optimal portfolio choice of the investor is thus the pointwise solution  $\omega_t^*$ .
- In addition, writing  $\mathbf{1}_n$  a vector of ones of size n, market clearing condition writes:  $\forall t, \ \omega_t^* = \mathbf{1}_n$ .
- Equilibrium expected returns are therefore  $\mu_t = \gamma \Sigma \mathbf{1}_n \beta E_t + \alpha M_t$ .

- 1. Show that, at optimum, optimal strategies verify the following:  $\kappa_c^i c_t^{i*} + \kappa_r^i r_t^{i*} = \frac{\beta}{\delta}$ .
- 2. Reduce the dimension of the problem by a change of variable:
  - ▶ State variables:  $(E, V, M) \Rightarrow (X, M), X := E V$  (overrating)
  - Controls:  $(c, r) \Rightarrow \xi, \quad \xi := c r$  (greenwashing effort)
  - Equivalent program:

$$\sup_{\substack{\xi=c-r,\\(r,c)\in\mathbb{A}}}\mathbb{E}\left[\int_0^\infty e^{-\delta t}\left(\beta X_t^{\mathsf{X}}-\alpha M_t^{\mathsf{u}}-\frac{\bar{\kappa}}{4}\left(\xi_t+\frac{\beta}{\delta\kappa_r}\right)^2\right)dt\right].$$

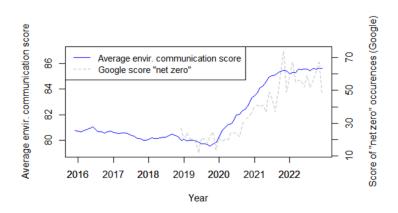
3. Solve the equivalent program with **one-dimensional** control variable. HJB equation:

$$\max_{\xi \in \mathbb{R}} \left\{ \beta x - \alpha u - \frac{\bar{\kappa}}{4} \left( \xi + \frac{\beta}{\delta \kappa_r} \right)^2 - \delta v + \frac{\partial v}{\partial x} (-ax + \xi) - \frac{\partial v}{\partial u} \rho u + \frac{z^2}{2} \frac{\partial^2 v}{\partial x^2} + \lambda \left[ v(x(1-b), u + b^2 x^2) - v(x, u) \right] \right\} = 0.$$

4. Deduce optimal strategies in the optimal problem using equality stated in 1.

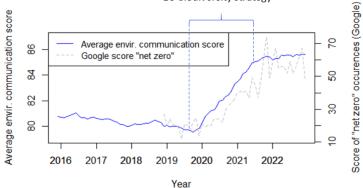


## Estimation: Step 1 (Environmental communication)

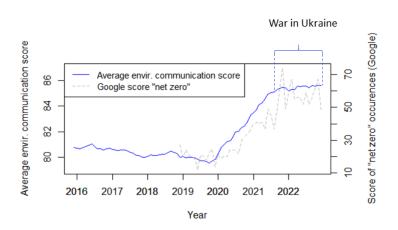


Key environmental regulations worldwide. E.g., EU:

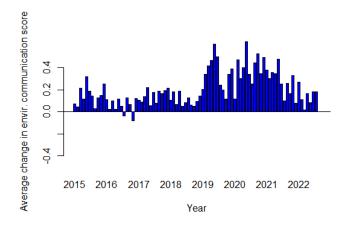
- EU Green Deal, regulations on binding annual emission reductions,
- circular economy,
- sustainable finance,
- EU biodiversity strategy



### Estimation: Step 1 (Environmental communication)



# Estimation: Step 1 (Environmental comm effort, $\hat{c}_t^i$ )



 $\Rightarrow$  98.8% of the average monthly environmental communication over the period is positive.

Dependent variable:  $\Delta \hat{c}^i$ 

# Estimation: Step 2 ( $\Delta \hat{c}_t^i = \alpha_3^i + \iota_{3,t} + \beta_3 \Delta E_t^{i,*} + \varepsilon_{3,t}^i$ )

	Dependent variable. $\Delta e_t$							
	Top brownest companies:							
	10%	20%	30%	40%	50%			
$\Delta E_t^{i,*}$	-0.071	-0.164**	-0.244***	-0.221***	-0.271***			
·	(0.051)	(0.065)	(0.073)	(0.067)	(0.060)			
Firm FE	Yes	Yes	Yes	Yes	Yes			
Month FE	Yes	Yes	Yes	Yes	Yes			
Observations	18,760	30,711	44,116	56,785	68,276			
$\mathbb{R}^2$	0.005	0.006	0.008	0.010	0.013			
Adjusted R <sup>2</sup>	$1 R^2 -0.061 -0.0$		-0.041	-0.035	-0.029			
F Statistic	0.985	3.525*	5.460**	3.608*	4.949**			
	Dependent variable: $\Delta \hat{c}_t^i$							
		Top	brownest cor	npanies:				
	60%	70%	80%	90%	Whole sample			
$\Delta E_{t}^{i,*}$	-0.237***	-0.176***	-0.188***	-0.158***	-0.119***			
·	(0.053)	(0.049)	(0.046)	(0.040)	(0.033)			
Firm FE	Yes	Yes	Yes	Yes	Yes			
Month FE	Yes	Yes	Yes	Yes	Yes			
Observations	83,309	97,324	110,206	123,864	145,508			
$\mathbb{R}^2$	0.015	0.016	0.017	0.017	0.017			
Adjusted R <sup>2</sup>	-0.023	-0.019	-0.015	-0.012	-0.008			
F Statistic	3.476*	1.756	1.875	1.195	0.661			

Note:

Dependent variable:  $\Lambda \hat{c}^i$ 

# Estimation: Step 2 ( $\Delta \hat{c}_t^i = \alpha_3^i + \iota_{3,t} + \beta_3 \Delta E_t^{i,*} + \varepsilon_{3,t}^i$ )

	Dependent variable: $\Delta c_t$							
	Top greenest companies:							
	10%	20%	30%	40%	50%			
$\Delta E_t^{i,*}$	-0.255***	-0.342***	-0.446***	-0.405***	-0.415***			
	(0.079)	(0.069)	(0.072)	(0.061)	(0.057)			
Firm FE	Yes	Yes	Yes	Yes	Yes			
Month FE	Yes	Yes	Yes	Yes	Yes			
Observations	21,644	35,302	48,184	62,199	77,232			
$\mathbb{R}^2$	0.018	0.019	0.021	0.020	0.020			
Adjusted R <sup>2</sup>	-0.018	-0.018 $-0.013$ $-0.01$		-0.010	-0.009			
F Statistic	4.284**	8.542***	14.584***	11.377***	10.606***			
	Dependent variable: $\Delta \hat{c}_t^i$							
	Top greenest companies:							
	60%	70%	80%	90%	Whole sample			
$\Delta E_{t}^{i,*}$	-0.404***	-0.380***	-0.294***	-0.237***	-0.119***			
ι	(0.052)	(0.054)	(0.052)	(0.044)	(0.033)			
Firm FE	Yes	Yes	Yes	Yes	Yes			
Month FE	Yes	Yes	Yes	Yes	Yes			
Observations	88,723	101,392	114,797	126,748	145,508			
$\mathbb{R}^2$	0.022	0.022	0.022	0.021	0.017			
Adjusted R <sup>2</sup>	-0.007	-0.006	-0.006	-0.006	-0.008			
F Statistic	8.727***	6.709***	3.513*	2.169	0.661			

Note:

### Testing the equation of optimal communication

⇒ Companies, **especially the greenest ones**, use **environmental communication** in a **counter-cyclical way** with respect to the evolution of their environmental score, in line with the results of the model.

#### The results are **robust** to:

- Controling for systematic risks and returns.
- Repeating the estimation starting at different dates: December 2012, December 2017, December 2019, and December 2021.
- Using 3 environmental subscores related to (i) the environmental impacts of the products sold, (ii) the resources used, and (iii) the emissions, effluents, and waste.

# What about greenwashing?

#### **Conclusions** about environmental communication:

- 1. Companies have implemented a quasi-structural positive envir. com. policy
- 2. Counter-cyclical dynamic of the envir. com., as highlighted by the model

#### Three possible interpretations:

- 1. Companies are structurally underrated.
  - $\rightarrow$  But no evidence of underrating; in addition evidence that rating agencies tend to be biased in favor of borrowers (Manso, 2013)
- 2. Companies use communication to support their continuous abatement effort.
  - ightarrow But monthly communication is very likely to be more volatile than environmental value.
- 3. Companies **greenwash** at least part of the time.
  - $\rightarrow$  Supported by the low MUC of communication and the asymmetry of information (Barbalau and Zeni, 2023).
- ⇒ The **greenwashing** option, at least part of the time, is the most likely.

#### Robustness: Controls

10% -0.205 (0.182) -0.335 (0.287) 0.005 (0.015) Yes Yes	Top 20% -0.380** (0.178) -0.222 (0.245) 0.008 (0.014) Yes	9 greenest com 30% -0.261* (0.142) -0.002 (0.217) -0.013 (0.027)	40% -0.243** (0.096) 0.348 (0.241) 0.008 (0.013)	50% -0.280*** (0.093) 0.480** (0.232) -0.009		
-0.205 (0.182) -0.335 (0.287) 0.005 (0.015) Yes Yes	-0.380** (0.178) -0.222 (0.245) 0.008 (0.014)	-0.261* (0.142) -0.002 (0.217) -0.013	-0.243** (0.096) 0.348 (0.241) 0.008	-0.280*** (0.093) 0.480** (0.232)		
(0.182) -0.335 (0.287) 0.005 (0.015) Yes Yes	(0.178) -0.222 (0.245) 0.008 (0.014)	(0.142) -0.002 (0.217) -0.013	(0.096) 0.348 (0.241) 0.008	(0.093) 0.480** (0.232)		
-0.335 (0.287) 0.005 (0.015) Yes Yes	-0.222 (0.245) 0.008 (0.014)	-0.002 (0.217) -0.013	0.348 (0.241) 0.008	0.480** (0.232)		
(0.287) 0.005 (0.015) Yes Yes	(0.245) 0.008 (0.014)	(0.217) -0.013	(0.241) 0.008	(0.232)		
0.005 (0.015) Yes Yes	0.008 (0.014)	-0.013	0.008			
(0.015) Yes Yes	(0.014)			-0.009		
Yes Yes		(0.027)	(0.012)			
Yes	N.		(0.013)	(0.014)		
		Yes	Yes	Yes		
	Yes	Yes	Yes	Yes		
8,084	12,272	16,003	19,503	23,219		
0.016	0.021	0.023	0.022	0.020		
-0.023	-0.012	-0.008	-0.009	-0.009		
1.504	3.582	1.748	3.120	5.449		
Dependent variable: $\Delta \hat{c}_t^i$						
	Top	greenest con	panies:			
60%	70%	80%	90%	Whole sample		
-0.385****	-0.284***	-0.251***	-0.193***	-0.083*		
(0.093)	(0.086)	(0.093)	(0.067)	(0.050)		
0.375*	0.185	0.316*	0.255*	0.252**		
(0.220)	(0.170)	(0.171)	(0.153)	(0.124)		
0.005	0.008	-0.011	-0.0002	0.010		
(0.011)	(0.011)	(0.012)	(0.010)	(0.007)		
Yes	Yes	Yes	Yes	Yes		
Yes	Yes	Yes	Yes	Yes		
25,745	28,779	32,062	35,208	41,252		
0.023	0.022	0.023	0.022	0.016		
-0.007	-0.007	-0.006	-0.006	-0.012		
	2.722	4.029	2.754	3.014		
	-0.385*** (0.093) 0.375* (0.220) 0.005 (0.011) Yes Yes 25,745 0.023	Top 70% 70% 70% 70% 70% 70% 70% 70% 70% 70%	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		

#### Robustness: Period

		Dependent	variable: $\Delta \hat{c}_t^i$			
		50% browne	st companies			
	Since $2012$	Since 2017	Since 2019	Since 2021		
$\Delta E_t^{i,*}$	$-0.271^{***}$ $(0.060)$	$-0.226^{***}$ $(0.057)$	$-0.220^{***}$ $(0.072)$	-0.237*** $(0.087)$		
Firm FE Time FE	Yes Yes	Yes Yes	Yes Yes	Yes Yes		
Observations R <sup>2</sup> Adjusted R <sup>2</sup> F Statistic	68,276 $0.013$ $-0.029$ $4.949**$	$57,626$ $0.014$ $-0.034$ $3.497^*$	$43,107$ $0.019$ $-0.042$ $3.420^*$	19,098 0.022 -0.093 4.817**		
	Dependent variable: $\Delta \hat{c}_t^i$					
	Since 2012	50% greenes Since 2017	st companies Since 2019	Since 2021		
$\Delta E_t^{i,*}$	-0.415*** (0.057)	$-0.457^{***}$ $(0.061)$	-0.449*** (0.065)	-0.353*** (0.069)		
Firm FE Time FE	Yes Yes	Yes Yes	Yes Yes	Yes Yes		
Observations R <sup>2</sup> Adjusted R <sup>2</sup> F Statistic	77,232 0.020 -0.009 10.606***	64,719 0.022 -0.012 13.629***	48,000 0.026 -0.020 18.549***	20,768 0.029 -0.075 9.557***		

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

#### Robustness: Subscores

	Depe	ndent variable	$\Delta \hat{c}_t^i$		Deper	ndent variable	$\Delta \hat{c}_t^i$
	50% brownest companies		-	50% greenest companies			
	(1)	(2)	(3)		(1)	(2)	(3)
$\Delta E_t^{Imp,i,*}$	-0.142*** (0.046)			$\Delta E_t^{Imp,i,*}$	-0.269*** $(0.042)$		
$\Delta E_t^{Res,i,*}$		$-0.180^{***}$ $(0.047)$		$\Delta E_t^{Res,i,*}$		-0.252*** $(0.038)$	
$\Delta E_t^{Emi,i,*}$			$-0.204^{***}$ $(0.051)$	$\Delta E_t^{Emi,i,*}$			-0.225*** $(0.036)$
Firm FE	Yes	Yes	Yes	Firm FE	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Time FE	Yes	Yes	Yes
Observations	68,276	68,276	68,276	Observations	77,232	77,232	77,232
$\mathbb{R}^2$	0.006	0.005	0.015	$\mathbb{R}^2$	0.013	0.009	0.014
Adjusted R <sup>2</sup>	-0.036	-0.037	-0.027	Adjusted R <sup>2</sup>	-0.016	-0.020	-0.016
F Statistic	2.087	3.580*	3.978**	F Statistic	5.953** 72	8.354***	8.135***
				Note:	*p<	<0.1; **p<0.05	5; ***p<0.01



#### Directional marginal benefits

Let  $\epsilon > 0$ . For a pair of communication and abatement strategies  $c, r \in \mathbb{A}$  and a pair of test functions  $\delta c, \delta r \in \mathbb{A}$ , let us define the associated pair of modified strategies:

$$c_s^{\epsilon} := c_s + \epsilon \delta c_s, \qquad r_s^{\epsilon} := r_s + \epsilon \delta r.$$

Define the functional J(c, r) as the expected discounted integral of the cost of capital:

$$J(c,r) := \mathbb{E}\left[\int_0^\infty e^{-\delta t} \left\{-\gamma \Sigma \mathbf{1}_n + \beta E_t^{c,r} - \frac{\alpha}{\alpha} M_t^{c,r}\right\} dt\right],$$

Then, the expected marginal benefits of communication and abatement along directions  $\delta c$  and  $\delta r$  are defined respectively as the directional (Gateaux) derivatives of J in these two directions:

$$\lim_{\epsilon \to 0} \frac{1}{\epsilon} \left( J(c + \epsilon \delta c, r) - J(c, r) \right), \qquad \lim_{\epsilon \to 0} \frac{1}{\epsilon} \left( J(c, r + \epsilon \delta r) - J(c, r) \right).$$



#### Marginal benefits of emissions reduction and communication

The directional marginal benefits (Gâteaux derivatives) are linear, and can be expressed through Frechet derivatives  $D_t^c$  and  $D_t^r$ :

$$\lim_{\epsilon \to 0} \frac{1}{\epsilon} \left( J(c + \epsilon \delta c, r) - J(c, r) \right) = \mathbb{E} \left[ \int_0^\infty e^{-\delta t} D_t^c J(c, r) \, \delta c_t \, dt \right],$$

$$\lim_{\epsilon \to 0} \frac{1}{\epsilon} \left( J(c, r + \epsilon \delta r) - J(c, r) \right) = \mathbb{E} \left[ \int_0^\infty e^{-\delta t} D_t^r J(c, r) \, \delta r_t \, dt \right].$$

The derivatives  $D_t^c$  and  $D_t^r$  shall be called marginal benefits of increasing communication or abatement at a given time t.



#### Reference calibration

Table: Calibration.

Parameter	Value
а	0.4
b	1
$\lambda$	8.5%
$\kappa_{m{c}}$	1
$\kappa_r$	50
$oldsymbol{eta}$	1
$\alpha$	1
ho	0.1
$\delta$	0.1
Z	0.2

