Financial Market Structure for ESG Integration

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Abstract

We explore financial market structures that incentivize firms to prioritize ESG. Borrowers may prefer reducing expected interest payments by pursuing ESG over financial profits, particularly at high borrowing rates. However, competition between ESG-friendly and non-ESG lenders lowers equilibrium borrowing rates, discouraging ESG prioritization. When firms privately know their true ESG preferences, early-moving ESG-friendly lenders can "cleanse" the ESG capital market. Specifically, non-ESG lenders perceive holdout borrowers as those with strong ESG preferences and thus charge high borrowing rates so that even non-ESG borrowers pursue ESG to reduce interest payments. Ultimately, promoting lender competition may deter rather than support ESG integration.

Keywords: Socially responsible investments, ESG, moral hazard, adverse selection, financial market structure, greenwashing

JEL Codes: D82, D86, G23, G31

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1 Introduction

ESG is too big of a phenomenon to be dismissed. As of 2022, 98 percent of S&P 500 firms issue ESG reports and claim that ESG is an integral part of their business strategy and important long-term investment (G&A Institute, 2023). In addition, total asset under management of United Nations Principles of Responsible Investment (PRI) signatories who committed to incorporate ESG into portfolio decision has reached $121 trillion (UN PRI, 2024). However, there is a growing concern in the ESG capital market that those allocated with the capital may not "walk the talk" on their ESG promises (Reuters, 2020; SEC, 2022*a*). Particularly, the lack of legally binding accounting standards on ESG, which could guide managers and investors on how to measure firms' ESG performance, makes most claims on ESG hardly verifiable, and therefore, hampers investors from properly monitoring corporate issuers' ESG integration (SEC, 2022*b*; Bebchuk and Tallarita, 2020).[1] Against this backdrop, this paper theoretically identifies the financial market structure that can facilitate ESG integration. At a high level, we find that promoting competition between lenders may not always facilitate ESG integration by borrowers. Indeed, a competitive lending market prevents incumbent ESG-friendly lenders from "cleansing" the ESG capital market.

In our model, a firm can undertake one of two investment projects: *(i)* the ESG-friendly (henceforth "green") project that yields a binary stochastic financial return with a low probability of success while generating social return realized in the form of externality, or *(ii)* the non-ESG (henceforth "brown") project that yields a binary stochastic financial return with a high probability of success but zero social return. The firm is assumed to value both net financial income and social return from its investment but with different weights. The firm is further assumed to be cashless, so it must borrow from one of the outside lenders to finance its project with a promise to repay, contingent on the success of its investment. Two different groups of lenders compete to finance the firm's project. One group of lenders is the "green" type, which equally values the financial income and the externality generated by the firm's investment. The other group of lenders is the "brown" type that cares only about the financial payoff.

A crucial assumption in our model is that no lender can attach a binding covenant to her financial contract that restricts the borrowing firm's investment decision (i.e., the firm cannot commit to its project selection *ex-ante*). That is, lenders can offer a contract only

[1] ESG integration herein can be broadly defined as borrowers' ex-post following-through on ESG (or engagement in ESG) as promised when they borrow the funds from lenders *ex ante*.

with a financial repayment term — an amount of the financial return promised to be repaid. Our assumption incorporates the idea that claims of ESG investment practices are difficult to verify due to the lack of a disclosure mandate for such ESG claims. This unverifiability of ESG claims can easily lead to inaction (or greenwashing), as empirically documented by Baker et al. (2023), Gibson Brandon et al. (2022), Kim and Yoon (2023), Liang, Sun and Teo (2022), and Raghunandan and Rajgopal (2022*a*). Under our assumption, the repayment amount is the only contractual instrument influencing the firm's incentive to make ex-post project selections. Indeed, the most favorable contract offer to the firm will be the one that charges the smallest repayment.

We find that the firm has an incentive to choose the green project only if the repayment term is sufficiently high. The intuition is twofold. First, the firm can effectively save the total expected repayment by shifting risk to the green project, which has a smaller probability of success and thus fulfilling the repayment obligation. This benefit of lowering the borrowing cost increases with the repayment amount, thereby incentivizing the firm to save the repayment by choosing the green project (Chang, Rhee and Yoon, 2024). Second, the firm endogenously values social return more than the financial payoff from its investment as the repayment increases. This is because a large portion of the financial return will be paid out to the lender regardless of the likelihood of the firm's project generating a high financial return (Chowdhry, Davies and Waters, 2019). In this regard, a high financial repayment term effectively aligns the incentives between the firm and green lenders.

We further find that the competition with brown lenders deters, rather than facilitates, green investments. Suppose there are only green lenders in the capital market first. Recall that green lenders fully incorporate the externality of the firm's investment into their payoffs. Thus, any green lender, even if she deviates and offers a contract to induce brown investment, must charge a "dirty" premium for forgoing the positive externality of green investments. This "dirty" premium attached to the financial repayment raises the cost of borrowing from the deviating green lender, leading the firm either to reject the deviation offer outright or to accept it but choose the green project.

However, if present in the market, brown lenders would be willing to charge a small repayment as long as the firm, in response, is believed to choose the brown project with a small default risk. Under the competition with brown lenders, green lenders cannot effectively raise the firm's borrowing cost in equilibrium. Indeed, the firm strictly prefers lowering the cost of borrowing no matter which of the two projects it selects *ex-post*. Thus, the firm will reject any offer with a financial repayment larger than the brown lenders' offer, resulting in brown investments.

Nevertheless, green lenders can play a crucial role in achieving ESG integration in the presence of adverse selection on the firm's preference for social value. Specifically, we extend our model by assuming that the firm can be of two types: *(i)* the "green" firm that values the social return from its investment, or *(ii)* the "brown" firm that places a smaller weight on the social return than the green type. Only the firm privately knows its type, which can create an adverse selection problem in the capital market. In this extension, a key question is whether and how the brown firm is incentivized to choose the green project. Particularly, we analyze the role of competition structure in ESG integration. In this context, we consider some green lenders place bids to finance first, whereas the rest of the lenders subsequently offer only after the firm rejects all offers from the first-moving green lenders.[2] In such a structure, the firm's rejection to borrow in the earlier market may endogenously provide additional information to the later lenders about the firm's inherent preference for social value.

An endogenous posterior belief formation naturally leads to multiple equilibria. Among them, we find an equilibrium in which the brown firm with certainty invests in the green project. In this equilibrium, the first-moving green lenders "cleanse" the capital market by funding, and thus taking out, the brown firm first. By doing so, the holdout firm that does not borrow in the earlier market is likely to be the green type. The subsequent brown lenders then require a large financial repayment because they anticipate the holdout firm will choose the green project with a high financial risk. This large repayment in turn incentivizes the firm, regardless of its type, to select the green project. That is, the first-moving green lenders cleanse the subsequent market by leaving the green firm as the likely borrower to whom brown lenders provide capital. Furthermore, the green lenders in the earlier market rationally infer that the brown firm cannot finance its project cheaply in the subsequent market after rejecting to borrow in the earlier market. Hence, it is optimal for the green lenders in the earlier market to offer the same hefty financial repayment as the subsequent brown lenders. Thus, the brown firm must borrow from the first-moving green lenders with a large repayment obligation and choose the green project correspondingly for the purpose of saving the repayment. In sum, green lenders cleanse the earlier capital market by raising the overall borrowing cost of the brown firm, eventually incentivizing the brown firm to choose the green project.

[2] We note that this is a very plausible scenario. For example, the Japanese Government Pension Investment Fund (GPIF), the world's largest pension fund and advocate of ESG, has been allocating only to asset managers and firms that incorporate ESG. Given their market power and existing relationships with borrowers, borrowers are likely to seek out the GPIF again to finance their projects.

Currently, many regulators are considering potential ESG mandates and we are in the nascent stages of understanding the impact (Krueger et al., 2021). There are challenges, because the scope of ESG disclosure is not agreed upon and is in the nascent stages of being verified (Christensen, Hail and Leuz, 2021; Gipper, Ross and Shi, 2022; Serafeim and Yoon, 2023). As a result, the costs and benefits of regulating ESG performance are not well assessed. Against this backdrop, our analysis has an important policy implication for facilitating ESG integration. First, promoting competition between brown and green lenders must be accompanied by properly monitoring the firms' follow-through. Particularly, in the presence of brown lenders, competition between green and brown lenders will lower the overall borrowing cost, which may adversely incentivize firms to behave privately against the interest of green lenders. Second, given that some green lenders hold an incumbency advantage in the capital market (e.g., the case of Japanese GPIF), granting fair competition opportunities to new entrant lenders may weaken the role of the incumbent green lenders in "cleansing" the market. Such policies can adversely reduce the efficiency of resource allocation to ESG integration.

The remainder of this paper proceeds as follows. Section 2 presents our model and Section 3 analyzes a baseline case. In Section 4, we provide equilibrium analysis in adverse selection. Section 5 discusses related literature and Section 6 concludes. Proofs are deferred to the Appendix.

2 Model

Consider an economy consisting of a firm and lenders. Lenders make contract offers to fund the firm's financial project. The firm accepts the most favorable offer and then chooses a financial project. At the game's end, the firm's project generates returns, which will accrue to the lenders and the firm according to the financial contract.

The firm can undertake a financial project that generates financial and social returns. Specifically, the firm can undertake either "green" (indexed by G) or "brown" (indexed by B) project.[3] Both projects require a unit of initiation cost paid in financial wealth. However, the firm is assumed to be endowed with zero wealth, so it must externally finance its project by borrowing from outside lenders. If the firm invests in the green project, it generates a positive financial return R with probability $p_G \in (0,1)$ and nothing with probability $1 - p_G$.

[3] Our primary focus is a potential incentive (mis-)alignment between the firms who conduct capital budgeting of new ESG projects and new outside lenders who finance the firms' new projects. For analytical simplicity, we abstract an agency problem within the firm from the current paper.

The investment in the green project also generates a non-financial positive externality $\phi > 0$, which refers to the social benefit from ESG integration. If the firm invests in the brown project, it generates the same positive financial return R with probability $p_B \in (0, 1)$ and nothing with probability $1 - p_B$. However, unlike the green project, the brown project does not generate any social return.

For the relevance of the analysis, we assume the following:

Assumption 1. *(i)* $p_G < p_B$*; (ii)* $p_G R \geq 1$*.*

Assumption 1-(i) means that the net present (financial) value (NPV) of the brown project is strictly higher than the green project. Assumption 1-(ii) means that both projects are non-negative NPV. This assumption is introduced to rule out a trivial outcome in which the green project is not funded simply because of a negative NPV. Furthermore, this assumption is also consistent with papers such as Edmans (2011), Khan, Serafeim and Yoon (2016), and Bolton and Kacperczyk (2021) that find certain ESG investments can enhance shareholder value.

We further assume that the firm is risk neutral, and its utility is a linear weighted sum of net financial income and externality. Formally, the firm's utility is calculated by adding the externality of the firm's investment with weight $\lambda \geq 0$ to the firm's net financial income. The parameter λ herein represents the firm's innate preference for the non-financial social return of its new investment project during the capital budgeting process. The higher λ, the more the firm cares about the social consequences of the new investment than financial profit.

This assumption of the firm's innate preference for social value can be justified by the previous studies that find socially responsible investors willingly forgo some expected financial returns for social or moral considerations (Renneboog, Ter Horst and Zhang, 2008; Barber, Morse and Yasuda, 2021). Another strand of literature documents how corporate executives financially benefit from pursuing sustainability in the long run. For example, active community engagements can bring favorable regulatory, legislative, and fiscal treatments by the local government (Berman et al., 1999; Freeman, 2010; Hillman and Keim, 2001; Waddock and Graves, 1997). Furthermore, improving employee benefits enhances the productivity and morale of current employees and increases the likelihood of recruiting talent (Turban and Greening, 1997). Moreover, corporate philanthropy is an effective business strategy to build a good reputation towards customers and stakeholders (Freeman, Harrison and Wicks, 2007; Fombrun and Shanley, 1990).

Each lender is endowed with a unit of financial wealth and competes to finance the firm's project in the capital market. When the market opens, the lenders make contract offers to the firm. The firm borrows from a lender who makes the most favorable offer. If multiple lenders are making the same offer that the firm finds the most favorable, these lenders are equally likely to be chosen by the firm.[4] The lenders are risk-neutral and classified into two types: "green" and "brown." First, there is a finite but large number of brown lenders who do not have inherent preferences for the externality generated by the firm's project.[5] That is, brown lenders are akin to conventional lenders in a standard corporate finance model whose utility is equal to the net expected financial payoffs. In contrast, there is also a finite but large number of green lenders who heavily consider the social impact of the firm's investment. Specifically, the utility of green lenders is a sum of the financial payoff and the externality of the firm's investment with equal weights.

There are several assumptions about the financial market structure. First, the externality of the firm's investment is added to the utility of each green lender, regardless of whether she funds the firm's project herself.[6] Thus, when making an offer to the firm, each green lender must consider *(i)* how much financial payoff she will get and *(ii)* whether she can increase the externality of the firm's investment by financing the firm's project directly.

Second, no lender can attach any binding covenant to her contract offer that enforces the firm's ex-post project selection. We assume that a financial repayment obligation D is the only term that each lender can offer to the firm. Since the firm's project yields zero return when unsuccessful, the lenders can offer how much they will get repaid conditional on the high financial return of the firm's project.[7] The assumption above is introduced to incorporate the greenwashing problem into our model. Since it is impossible for any lender

[4]Specifically, we assume that the firm does not observe the identities of the lenders but the offers only. That is, the firm's strategy of accepting an offer is not conditional on the lenders' types or identities. Formally, suppose that $n \geq 2$ lenders have made the offer. If the firm decides to accept this offer, the firm ends up contracting with each of them with probability $1/n$. We interpret such randomization as a move of a chance node rather than the firm's voluntary randomization.

[5]The assumption of finitely many lenders is introduced primarily for analytical convenience. Indeed, the finiteness of the lenders can rule out any perverse equilibrium in which lenders make a contract offer yielding a negative payoff ex-post. However, the equilibrium analysis remains unchanged even if there are infinitely many lenders in the model by adding the assumption that any lender, after winning the competition, has the option to default on funding if performing the contract obligation is expected to yield a net loss (Board, 2007).

[6]In other words, the green lenders in our model are viewed as *consequentialists* in their evaluation of the social impact of the firm's investment (Sinnott-Armstrong, 2023).

[7]This state-contingent repayment $D \in [0, R]$ is equivalently expressed as a borrowing rate $r \equiv D - 1$. Furthermore, under the assumption that the firm's project yields zero return when unsuccessful, there is no issue with foreclosure when the firm fails to fulfill its repayment obligation.

to earmark the proceeds she lends to the firm for a specific project, the firm will select the financial project that serves its private interest but possibly harms some, particularly green, lenders. We will analyze how the lack of a binding green-investing covenant influences the integration of green investments. With the assumption that the firm's project yields either positive or zero return, the firm finds no difference in whether lenders have debt or equity claims with the same repayment contingent on the high financial return (Tirole, 2010, p.119). This feature streamlines our analysis without specifying the securities the firm issues when funding its project.[8]

Moreover, we assume that all lenders simultaneously make contract offers *à la* Bertrand in the baseline model in Section 3. This assumption will be relaxed in the extension model introduced in Section 4 to analyze the impacts of various financial market structures on green investment.

Lastly, we assume the following regularity condition:

Assumption 2. $p_G R + \phi > p_B R$.

Assumption 2 means that the green project yields a strictly higher surplus to the green lenders than the brown project. Moreover, for analytical convenience, we assume the following choice rules in the event of indifference: the firm breaks a tie by investing in the brown project whenever it finds the green and brown projects indifferent; every lender, regardless of her type, breaks a tie by offering a contract that yields a higher probability of funding whenever she is indifferent between two offers.

3 Baseline Analysis

We first characterize the equilibrium in a baseline model in which all lenders fully know λ representing the firm's inherent preference for the social value of its investment. We first analyze how the amount of financial repayment influences the firm's incentive for project selection *ex post*. We next establish pure-strategy subgame perfect equilibrium, which we refer to as equilibrium in this section.[9] Note that all lenders with the same type will symmetrically

[8] Chang, Rhee and Yoon (2024) provide a more detailed analysis of the impacts of capital structure on corporate borrowers' incentive for ESG integration.

[9] Baye and Morgan (1999) and Kaplan and Wettstein (2000) show that the standard Bertrand competition with complete information may admit a mixed-strategy equilibrium that yields a non-zero equilibrium profit. However, as some of the assumptions for their analysis fail in our model, the necessary and sufficient condition for the existence of such a mixed-strategy equilibrium is not directly applicable to our setting.

offer the same repayment term that breaks them even in equilibrium since they compete in a Bertrand fashion.

3.1 The Firm's Incentive Conditions

Due to the lack of a binding covenant in project selection, only the financial repayment term will shape the firm's incentive for project selection. Let $D \in [0, R]$ be an arbitrary amount of repayment the firm is obligated to repay for borrowing from a lender. Then the firm gets the expected payoff $p_G(R - D) + \lambda\phi$ from the green project and $p_B(R - D)$ from the brown project. Thus the firm chooses the green project if and only if

$$p_G(R - D) + \lambda\phi > p_B(R - D).$$

Then, we can derive the following incentive condition for the firm's project selection.

Lemma 1. *Let D be the financial repayment the firm is obliged to the lender. Then, the firm chooses the green project if and only if $D > \overline{D}(\lambda)$, where $\overline{D}(\lambda)$ is defined as*

$$\overline{D}(\lambda) := R - \lambda \frac{\phi}{p_B - p_G}. \tag{1}$$

Lemma 1 highlights an important necessary condition for green investments: lenders must charge a large financial repayment when funding the firm's investment. Indeed, a large repayment incentivizes the firm to choose the green project. Facing a large repayment obligation, the firm can reduce its expected repayment by shifting risk, i.e., investing in a project with a smaller probability of high return than the other project. Because $p_G < p_B$, the firm can save the expected repayment by $(p_B - p_G)D$ by switching to the green project. Notably, the firm's incentive for risk-shifting increases with the repayment term D, i.e., the expected repayment steeply changes with the firm's project selection.[10] Chowdhry, Davies and Waters (2019) similarly argues that the firm primarily pays attention to the social impact of its investment as the lender requires a significantly large share of the financial return from the firm's investment. Specifically, the firm could get little net financial income with a large financial repayment obligation, leading the firm to focus on the externality of the project, which

[10] The economic relationship between capital structure and incentive structure of ESG integration is of interest itself. Chang, Rhee and Yoon (2024) adopts a security design framework from DeMarzo, Kremer and Skrzypacz (2005) and finds that equity financing achieves green investments most likely, while debt financing achieves green investments least likely.

the firm fully enjoys without distributing to the lender.[11,12] In a nutshell, a large financial repayment obligation increases the total benefit of pursuing social value.

Another interesting observation is that the firm always prefers a contract requiring a small financial repayment, no matter who offers and how large λ is.

Lemma 2. *The firm always prefers the offer that charges the smallest financial repayment.*

Namely, the firm's payoff is a strictly decreasing function of D as follows:

$$\max \{p_B(R - D), p_G(R - D) + \lambda\phi\}.$$

If a lender (particularly the green one) can influence the firm's investment decision, she must win the competition for lending by charging the smallest financial repayment. However, intense competition among lenders may force each lender to excessively lower her repayment term, possibly leading the firm to pursue the brown investment. In Section 3.2 below, we will show how the competition structure influences the equilibrium outcome.

It is also worth noting that the threshold value $\overline{D}(\lambda)$ in (1) is decreasing in λ. Specifically, the firm takes more social value into account as λ increases, so the firm will primarily focus on the social value rather than the financial return from its investment. Lastly, the green investment can be made only if $\lambda > 0$: if $\lambda = 0$, then $\overline{D}(\lambda) = R$, so there is no feasible contract that induces the firm to make green investments. In Section 3.2 below, we will analyze how the equilibrium is pinned down by λ.

Lastly, Lemma 1 and 2 hint an important policy implication: restricting competition between lenders can improve rather than obstruct ESG integration. That is, a less competitive capital market raises the overall borrowing cost, which indeed lowers the opportunity cost of pursuing social returns and thus incentivizes borrowing firms to allocate their resources to ESG.

[11] This may seem counterintuitive but it is not unique to our setting. In the non-profit sector, Hansmann (1980) finds that nonprofit employers offer salaries below those in the for-profit sector to attract those who are likely to pursue the social good. Furthermore, Bond and Glode (2014) highlights that highly skilled human capital often chooses to work in regulatory roles with lower pay than those in the banking sector due to their motivation to work in the public sector.

[12] Choi, Kim and Kim (2022) empirically finds that fund managers can signal their commitment to ESG by complying with costly self-regulations. Our analysis provides an additional interpretation that the total borrowing cost also influences the opportunity cost incurred by keeping the firms' pledge to ESG integration.

3.2 Equilibrium

We establish equilibrium by applying the incentive conditions we derived in Section 3.1. Indeed, any equilibrium is represented by the amount of financial repayment D^*: there should be no other contract offer $D' \neq D^*$ that strictly benefits any lender; of all contract offers, the firm gets the highest payoff from accepting D^*; given D^*, the firm selects the project that serves it best.

To characterize the equilibrium, it is essential to check how the lenders infer the firm's ex-post project selection and its feedback effect on the firm's ex-ante incentive for contract selection. Thus, the main question is how the lenders' belief about the firm's project selection is determined. The following statement reveals that the equilibrium belief is pinned down by $\overline{D}(\lambda)$ in (1), representing the firm's actual willingness to invest in the green project.

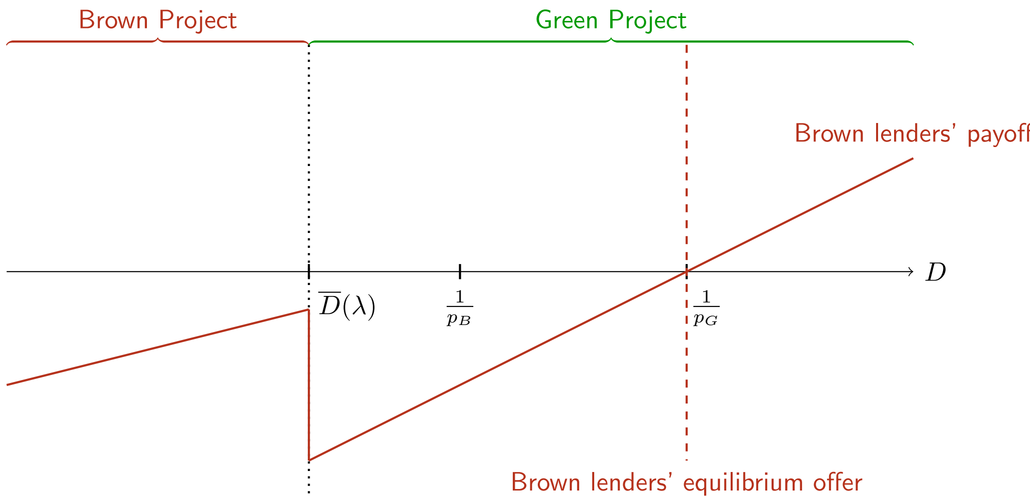
Theorem 1. *In every equilibrium, the equilibrium financial repayment D^* is uniquely determined. Specifically, the equilibrium is characterized as follows:*

(i) If $\overline{D}(\lambda) < \frac{1}{p_B}$, lenders offer $D^* = \frac{1}{p_G}$ and the firm invests in the green project;

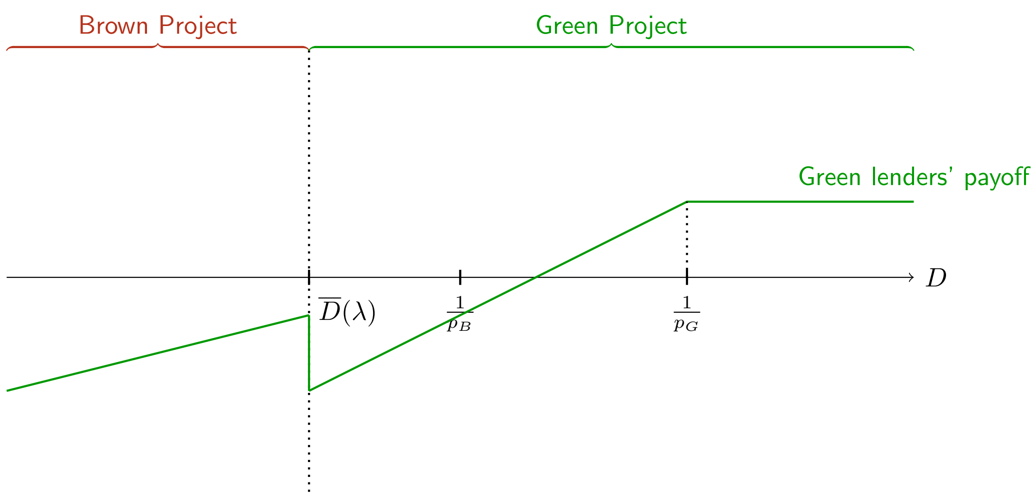
(ii) If $\overline{D}(\lambda) \geq \frac{1}{p_B}$, lenders offer $D^* = \frac{1}{p_B}$ and the firm invests in the brown project.

Suppose that $\overline{D}(\lambda)$ is relatively low, i.e., $\overline{D}(\lambda) < \frac{1}{p_B}$, first. Figure 1-(a) depicts the payoff of brown lenders as a function of the financial repayment D. First, brown lenders rationally believe that the firm borrowing from the brown lenders will choose the green project in equilibrium. To explain the intuition, suppose to the contrary that the firm is believed to choose the brown project in equilibrium. The Bertrand competition among the brown lenders yields the equilibrium offer $D = \frac{1}{p_B}$. However, such an offer cannot be the equilibrium strategy: the firm will choose the green project since $\frac{1}{p_B} > \overline{D}(\lambda)$, so the brown lender that actually finances the firm will suffer a financial loss $\frac{1}{p_B}p_G - 1 < 0$, which is a contradiction. Hence, with the belief that the firm will surely choose the green project, brown lenders expect that the project will yield a low expected financial return, so they require a large financial repayment $D^* = \frac{1}{p_G}$ correspondingly. Such a large financial repayment obligation in turn incentivizes the firm to choose the green project consistently.

Next, green lenders know that the firm will choose the green project if it borrows from brown lenders. Figure 1-(b) depicts the payoff of green lenders as a function of their offer D, given that brown lenders offer $D^* = \frac{1}{p_G}$. If green lenders want to fund the firm's project directly, they must offer a financial repayment term strictly lower than $D^* = \frac{1}{p_G}$.



(a) Brown lenders' payoff function



(b) Green lenders' payoff function

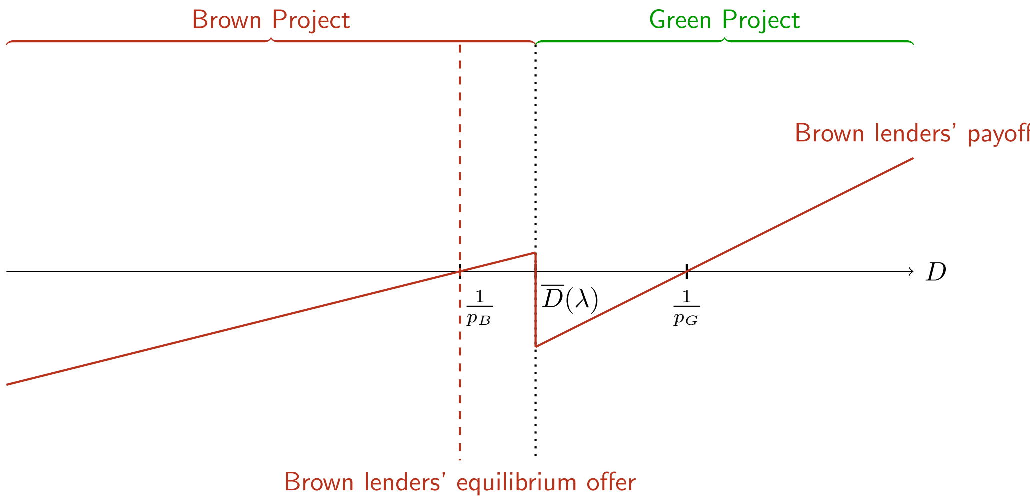
Figure 1 – Equilibrium payoff functions under $\overline{D}(\lambda) < \frac{1}{p_B}$

However, green lenders do not prefer playing such a strategy for two reasons. First, any financial repayment term $D' \in (\overline{D}(\lambda), D^*)$ will make a financial loss while generating the same externality as D^* would do, yielding a lower total payoff than offering D^*. Alternatively, if green lenders offer an excessively small financial repayment $D' \leq \overline{D}(\lambda)$, the firm will invest in the brown project that yields a strictly lower payoff to green lenders than offering D^*. Hence, green lenders have no incentive to offer a contract with a financial repayment strictly smaller than D^*, which supports the firm's project selection in equilibrium.

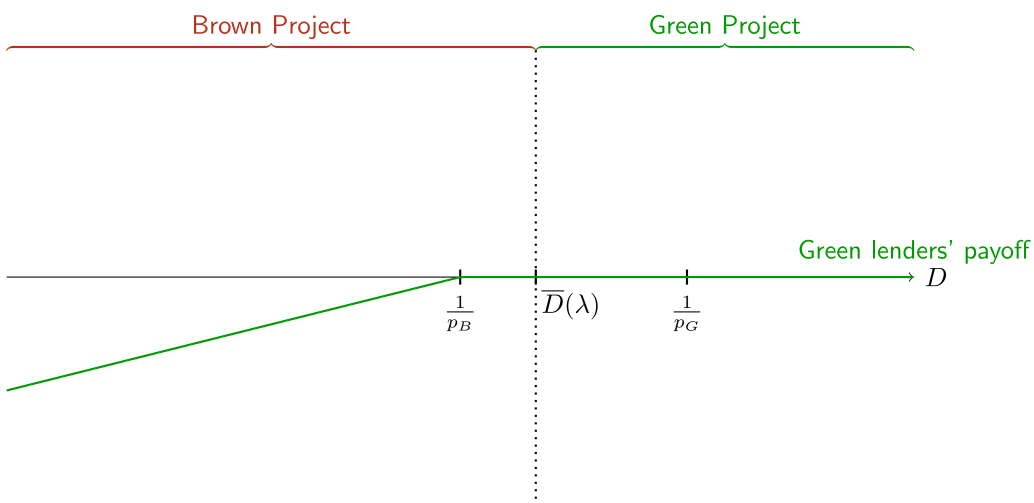
Next, suppose $\overline{D}(\lambda)$ is relatively high, that is, $\overline{D}(\lambda) \geq \frac{1}{p_B}$. Figure 2-(a) depicts the payoff of brown lenders as a function of the repayment term D. Then, brown lenders believe that the firm will choose the brown project if it borrows from the brown lenders. To explain why, suppose to the contrary that the firm chooses the green project. Then, the equilibrium offer must equal $\frac{1}{p_G}$, and the brown lenders, if they finance the firm, get the zero expected payoff. However, suppose one brown lender deviates and offers $D' \in [\frac{1}{p_B}, \overline{D}(\lambda)]$. In that case, the firm will accept such a deviating offer because it always prefers lowering the borrowing cost by Lemma 2. Furthermore, since $D' \leq \overline{D}(\lambda)$, the deviating offer will still induce the firm to choose the brown project, yielding a high financial return with a high probability. Therefore, the deviating brown lender can make a non-negative expected payoff from $D' \geq \frac{1}{p_B}$, which is a contradiction. Therefore, the firm borrowing from the brown lenders will choose the brown project. The brown lenders then offer a relatively low repayment term $D^* = \frac{1}{p_B}$ because the brown project generates a high expected financial return.

Figure 2-(b) depicts the payoff of green lenders as a function of their repayment term offer, given that brown lenders offer $D^* = \frac{1}{p_B}$. Green lenders know that the firm will choose the brown project if the green lenders do not make a strictly more favorable offer than the brown lenders. To influence the firm's investment decision in their favor, the green lenders must offer a repayment term $D' < \frac{1}{p_B}$. However, since such an offer is lower than $\overline{D}(\lambda)$, the firm will stick with the brown project even though it borrows from green lenders. Moreover, the green lenders will make a strict financial loss since $p_B D' < 1$. In sum, the green lenders find it optimal to offer a repayment term weakly greater than $D^* = \frac{1}{p_B}$, which supports the firm's project selection in equilibrium.

A noteworthy feature is that facing competition with brown lenders, green lenders cannot effectively influence the firm's investment decision without a binding covenant that enforces the project selection. Where possible, green lenders would like to offer a generously small loss-making financial repayment once the borrowing firm pledges to the green investment in exchange. In doing so, the green lenders can increase the investment surplus by preventing



(a) Brown lenders' payoff function



(b) Green lenders' payoff function

Figure 2 – Equilibrium payoff functions $\overline{D}(\lambda) \geq \frac{1}{p_B}$

the firm from borrowing from the brown lenders and choosing the brown project (Oehmke and Opp, 2024). However, as seen in Lemma 1, a small financial repayment will only adversely incentivize the firm to invest in the undesirable brown project when the project selection is not contractible. Instead, it is necessary for the financial repayment to be sufficiently high to provide the firm with the incentive to choose the green project. However, competition with brown lenders forces the equilibrium financial repayment to be small, possibly leading to the brown investment. The brown investment can be avoided only if the borrowing firm inherently appreciates the social return of its investment, which is referred to as a sufficiently low value of $\overline{D}(\lambda)$ (i.e., $\overline{D}(\lambda) < \frac{1}{p_B}$) in our analysis.

We next study whether the capital market structure influences the firm's investment decision. Specifically, we investigate whether the competition between lenders with different preferences for social value facilitates green investment. The following statement answers it does not.

Theorem 2. *Suppose that there are only green lenders in the market, who simultaneously compete in Bertrand fashion.*

(i) If $\frac{1}{p_B} \leq \overline{D}(\lambda) < \frac{1}{p_G}$, there exist two equilibria, one where the firm chooses the green project and the other where the firm chooses the brown project.

(ii) If $\overline{D}(\lambda) < \frac{1}{p_B}$, the firm chooses the green project in all equilibria.

(iii) If $\overline{D}(\lambda) \geq \frac{1}{p_G}$, the firm chooses the brown project in all equilibria.

Since $\overline{D}(\lambda)$ is decreasing in λ and $\phi > 0$, the equilibrium with green investment can arise from relatively low values of λ such that $\overline{D}(\lambda) \geq \frac{1}{p_B}$. Thus, Theorem 2-(i) suggests that the capital market with only green lenders can yield green investment even if the firm has a relatively weak preference for social value, so it would invest in the brown project when both green and brown lenders compete in the market. To explain the intuition clearly, suppose $\overline{D}(\lambda) < \frac{1}{p_G}$ and all green lenders offer a financial repayment $D_G = \frac{1}{p_G}$. Then a green lender can profitably deviate only by offering different terms of financial repayment $D' \leq \overline{D}(\lambda)$ that will induce brown investment. However, unlike brown lenders, the deviating green lender must add a "dirty" premium to the financial repayment as compensation for forgoing the high social value generated by the green investment (Bolton and Kacperczyk, 2021). This dirty premium will obviously raise the cost of borrowing. This increased cost of borrowing makes the deviation offer for brown investment infeasible: the firm will choose the green project even after accepting the deviating offer; the firm will reject the deviation offer outright due to the high cost of borrowing. This infeasibility of counteroffers that weakly better off the deviating green lender and induces brown investment sustains the equilibrium with green investment, even for relatively low values of λ, which would induce brown investment in the presence of competitive brown lenders.

Theorem 2 suggests an important policy implication: with the lack of institutional capability of monitoring borrowers' ESG claims, opening the capital market for ESG to brown lenders can deter rather than facilitate ESG integration. Given that brown lenders do not appreciate the social value generated by the firm's investment, they do not charge any dirty premium even though their contract offers may induce the brown investment. Indeed, brown lenders are willing to finance the firm at a significantly lower borrowing rate because such an offer yields brown investment with a smaller default probability. However, no effective contractual term is available that can help green lenders overcome weak price competence against brown lenders in the capital market. As a result, unless the firm has a sufficiently strong innate preference for social value, the firm cannot resist accepting brown lenders' offers with a low borrowing rate and then choosing the brown project.

Our result also differs from Oehmke and Opp (2024), which argues that competition between green and brown lenders enhances green investments. Their analysis concerns the case that green lenders can attach a binding covenant to their contract offers, which enforces green investment in exchange for cheap financing. Equipped with such a binding covenant, green lenders can give the firm a "clean" discount in exchange for green investment. Particularly, competition with brown lenders further incentivizes green lenders to give a more "clean" discount to the borrowing rate, facilitating green investment. On the other hand, our analysis shows that any competitive pressure that lowers the equilibrium borrowing rate deters green investment without the firm's commitment to its project selection. Therefore, before setting a direction for competitive policies in ESG credit markets, policymakers should carefully examine and find ways to facilitate the contractual enforceability of ex-post execution of ESG projects.

4 Adverse Selection and Green Investments

So far, we have assumed that the value of λ — representing the firm's inherent preference for the social value of its investment — is publicly known to all lenders. However, it is more natural to believe that only the borrowing firm privately knows how seriously it takes the long-term social consequences when making an investment decision. For example, Kim and Yoon (2023) finds that asset managers who claimed to engage in ESG integration attracted more fund flows but did not exhibit a follow-through increase in ESG performance. Furthermore, Cappucci (2018) and Kotsantonis and Serafeim (2019) point out that the lack of standards on ESG is a significant impediment to monitoring ESG integration. All these studies suggest that the adverse selection problem has not yet been appropriately addressed.

A pertinent research question is how the firm's investment decisions may change when lenders face an adverse selection problem regarding the firm's preference for social value. To address this question, we extend our model by assuming that the firm privately knows its innate preference for social value, which creates an adverse selection problem in the ESG capital market. In what follows, we first analyze how different competition structures can facilitate or hamper green investment in the presence of asymmetric information on the firm's preference for social value and then discuss policy implications derived from our analysis.

4.1 Competition Structures and Green Investments

We extend the baseline model by assuming that nature randomly draws λ from the set $\{\lambda_B, \lambda_G\}$ with a distribution function $Pr(\lambda = \lambda_G) = q \in (0, 1)$ before the financial market opens, where $\lambda_G > \lambda_B$. The firm privately knows the true value of λ. We refer to the realized value of λ as the firm's "type." Like in the baseline model, both green and brown lenders compete to finance the firm. To restrict our attention to the interesting cases, we assume the following regularity conditions:

Assumption 3. *(i)* $\frac{1}{p_G} > \overline{D}(\lambda_B) \geq \frac{1}{p_B} > \overline{D}(\lambda_G)$*; (ii)* $\overline{D}(\lambda_B) \geq \frac{1}{qp_G+(1-q)p_B}$.

Assumption 3-(i) has two features. First, the firm with type λ_G has a strong inherent preference for the social return to the extent that it would invest in the green project without adverse selection. In this context, the firm with type λ_G will be called the "green" firm. Second, the firm with type λ_B would choose the brown project without asymmetric information. In this respect, we will call the firm with type λ_B as the "brown" firm.[13] Furthermore, from

[13]In this section, our primary focus is on investigating if granting green lenders an incumbent competitive advantage *à la* Stackelberg helps achieve ESG integration (Theorem 3). Assumption 3-(i) is a necessary condition for the competitive advantage of green lenders to have any impact on the firm's project selection; if any inequality in Assumption 3-(i) fails, two versions of the model in Theorem 3 will yield the same equilibrium outcome. See Appendix C in the Online Appendix.

the feature that $\frac{1}{qp_G+(1-q)p_B}$ is increasing in q, Assumption 3-(ii) means that the lenders hold the prior belief that the firm is highly likely the brown type. This assumption is introduced to rule out the uninteresting cases: the firm, whichever type it may be, invests in the green project in equilibrium if the firm is believed to be of the green type with a high likelihood such that $\overline{D}(\lambda_B) < \frac{1}{qp_G+(1-q)p_B}$.[14]

A key question is how financial market structures can influence both firms' incentives to make green investments. To answer this, we modify the model by assuming that a subset of green lenders make contract offers to the firm before *all* brown lenders; the remaining lenders make contract offers in the later capital market only if the firm decides not to fund its project in the earlier capital market. Put differently, some (or possibly all) green lenders have a first-moving competitive advantage *à la* Stackelberg over brown lenders. The assumption of competitive advantage held by green lenders is also consistent with the notion that some ESG-endorsing institutional lenders had opened up this new capital market and thus maintained an incumbency advantage.[15] Next, we compare the equilibrium analysis results to those with an alternative model where some brown lenders compete to finance the firm's project *simultaneously* with the first-moving green lenders in the earlier market.

Lastly, we assume that lenders in the later capital market can only observe whether the firm accepts the offers made by early-bidding green lender(s) but cannot observe the exact terms of financial repayment that the firm accepts or rejects. This assumption is made primarily to simplify the analysis by shutting off the channels of dynamic belief updating. Nevertheless, this assumption is justified given that hedge fund and private equity firms, managing a sizeable amount of capital intended for ESG, privately negotiate fee structures (i.e., the additive inverse of repayment terms) with their clients before the other competitive borrowers.

We establish pure-strategy perfect Bayesian equilibrium, which we refer to as equilib-

[14] We provide the formal proof in Proposition C.4 in Appendix C in the Online Appendix.

[15] For example, several asset owners committed to ESG integration, such as the Japanese GPIF ($1.8 trillion), the Norwegian Sovereign Wealth Fund ($1.3 trillion), the Korea National Pension Fund ($766 billion), the California Public Employees' Retirement System (CalPERS) ($426 billion), and the California State Teachers Retirement System (CalSTRS) ($259 billion) (go to `https://www.pionline.com/interactive/worlds-largest-retirement-funds-2021` for more details). The total size of these funds is sizable compared to the entire US market capitalization, estimated at $ 40 trillion in 2022 (go to `https://siblisresearch.com/data/us-stock-market-value/` for more details). These owners already require asset managers to integrate the ESG metrics into their investment process and engage with their portfolio companies on critical ESG issues of their choosing (Henderson et al., 2019). Confirming this notion, Kim and Yoon (2023) finds that the Principles for Responsible Investment (PRI) signatories attracted significantly larger fund inflows from asset allocators after committing to incorporate ESG into their investment decisions.

rium hereafter.[16] We maintain the basic setups we introduced in Section 2. First, if the firm decides to accept an offer in either the earlier or the later market, the firm borrows from a lender who makes the most favorable offer. If multiple lenders are making the same offer that the firm finds the most favorable, these lenders are equally likely to be chosen by the firm. Next, we impose the same two tie-breaking rules: the firm breaks a tie by investing in the brown project whenever it finds the green and brown projects indifferent; every lender, regardless of her type, breaks a tie by offering a contract that yields a higher probability of funding whenever she is indifferent between two offers.

How does the presence of a first-moving green lender influence the equilibrium project selection of each firm type? The following partial characterization of the equilibrium outcome provides some clues.

Lemma 3. *Suppose that some green lenders make contract offers to the firm before all brown lenders. In any equilibrium, we have the following properties:*

(i) The green-type firm always chooses the green project with probability one;

(ii) Lenders in the later market assign a weakly higher probability to the firm rejecting the first-moving green lenders' offer as being of the green type than the prior belief, regardless of whether the firm's rejection of the early lending bid occurs on or off the equilibrium path.

Part (i) is immediate from Assumption 3-(i): the green firm has a sufficiently strong preference for social value so that it strictly prefers the green project no matter how high the equilibrium repayment is determined. Therefore, it suffices to analyze the brown firm's project selection in equilibrium.

Part (ii) states that, in any equilibrium, the later market has a more optimistic belief than the earlier market regarding the firm's inherent preference for social value. Suppose, to the contrary, that lenders in the later market assign a higher probability to the firm being brown and therefore charge a lower repayment term than the earlier market. Such a belief and the resulting lower repayment term in the later market could be rationalized only if the green firm borrows from a first-moving lender and exits the market early in equilibrium. However, the lower repayment term in the later market would attract both green and brown

[16] We adopt the definition of Perfect Bayesian equilibrium from Fudenberg and Tirole (1991, Section 8.2). In particular, consistent with the condition B-(iv) in Fudenberg and Tirole (1991), we impose that all lenders in the second period share the common posterior belief both on and off the path.

firms equally, contradicting the presupposition that the green firm exited earlier. In sum, the lenders in the later market must be (weakly) more optimistic about the firm's preference for social value than those in the earlier market.

Part (ii) has an important implication. The lenders in the later market rationally infer that the firm borrowing from them is likely to be green and thus likely to choose the financially under-performing green project. Hence, they would charge a (weakly) higher repayment than the early lenders do to the firm. In sum, the delayed borrowing signals the firm's strong preference for social value to the lenders in the later market. This would feed back into the borrowing term offered in the later market and shape the firm's timing of funding its project in equilibrium.

We now find an equilibrium that yields the green investment by the brown firm. Note that multiple equilibria naturally exist due to the endogenous formation of the posterior beliefs held by brown lenders; different beliefs about the firm contacting the brown lenders can give rise to different incentives for rejecting the green lenders' early offer. Nevertheless, the following statement reveals that the equilibrium inducing the green investment by the brown firm does exist under a certain financial market condition.

Theorem 3.

(i) *Suppose that green lenders make offers before brown lenders do. There are two types of equilibrium: an equilibrium where only the green firm invests in the green project with probability one, and another equilibrium where both green and brown firms invest in the green project with probability one. In the latter type of equilibrium, brown lenders in the later capital market believe the holdout firms rejecting the early offers are likely to be green type.*

(ii) *If green lenders no longer make offers before brown lenders do (i.e., at least one brown lender simultaneously bids with the first-moving green lender(s) in the early market), there exists no equilibrium in which both green and brown firms invest in the green project with probability one.*

Incumbent green lenders with the first-moving competitive advantage can play an important role in the investment that maximizes the social return (i.e., the brown firm chooses the green project) by lending to the firm ahead of the other lenders. This can be achieved with a belief formation by brown lenders in the late capital market such that the holdout firms rejecting the green lenders' offers — on or off the equilibrium path — are of the green type with a sufficiently high probability, say, close to one. With this belief, brown lenders expect that the borrowing firm will likely choose the green project and thus require a large financial repayment $\frac{1}{p_G}$, like in the baseline model absent adverse selection. Green lenders in the early financial market then have no incentive to offer any different terms of financial repayment strictly smaller than the later offer made by the brown lenders. Indeed, offering a small financial repayment will only lead to either a financial loss or an undesirable selection of the brown project (or both). Therefore, the lenders of both types find it optimal to offer the large financial repayment $\frac{1}{p_G}$ to the firm.

Furthermore, since all lenders offer the same financial repayment $\frac{1}{p_G}$ in both early and late capital markets, both green and brown firms are indifferent between accepting the green lenders' early offer and borrowing from the brown lenders' late offer. Hence, there can exist two possible equilibria, the one where only the brown firm accepts the early offer and the other where both firms accept the early offer. In the former equilibrium, the brown lenders' belief that the holdout firms are of the green type with a high probability becomes a posterior belief consistent with the firms' equilibrium strategies. In the latter equilibrium, the same belief as an out-of-equilibrium belief rationalizes the optimality of the firms' strategies. Nevertheless, the two equilibria induce both types of firms to make the same project selection on the equilibrium path. Lastly, by Lemma 1, both green and brown firms find it optimal to choose the green project at the large financial repayment term $\frac{1}{p_G}$.

The key feature of the financial market structure that can yield the highest social return is that some green lenders (and green lenders only) have an incumbent competitive advantage *à la* Stackelberg. A necessary condition for achieving green investment by the brown firm is that brown lenders must charge a sufficiently large financial repayment to the firm. In our setup, the only way to realize this outcome is that brown lenders hold a posterior belief that the holdout firm arriving in the later market is the green type with a sufficiently high likelihood. By making offers earlier than brown lenders, green lenders can "cleanse" the financial market through dynamic information revelation. For example, green lenders finance and thus take out the brown firm first, which makes the holdout firm approaching brown lenders perceived as the green firm with a high likelihood. Alternatively, both green and brown firms believe that rejecting the green lender's early bid signals their strong preference for social value. Either on or off the equilibrium path, brown lenders in the subsequent market expect the holdout firm to choose the green project and thus charge a large financial repayment in exchange for the low NPV of the green investment, inducing the actual green investment.

Interestingly, this anticipated cleansing effect on the subsequent financial market feeds back into the early financial market, inducing the firms to make green investments. Indeed, the firms expect it to be expensive to borrow from brown lenders in the subsequent market; the firm going to the subsequent market is believed to be the green type with a high likelihood. Green lenders in the early market rationally infer that the firms cannot get cheap financing from brown lenders. Hence, green lenders can also offer the same large financial repayment $\frac{1}{p_G}$ as brown lenders will do. Then, the firms have no choice but to accept this term of the large financial repayment and choose the green project afterward. Consequently, green lenders also cleanse the early market by endogenously raising the overall cost of borrowing of the firms in both early and subsequent financial markets.

The impact of sequential bidding on ESG integration is analogous to the public bailouts during financial crises to the extent that both can alleviate adverse selection problems. For instance, Philippon and Skreta (2012), Tirole (2012), and Che, Choe and Rhee (2023) argue that the government bailouts launched during financial crises effectively address the adverse selection problem and thus rejuvenate market lending by (endogenously) taking low-quality firms out of private markets. However, to entice these firms to the bailout programs, governments must withstand financial losses by offering undeniably generous terms. Similar to public bailouts, green lenders take (or are believed to take) the brown firm out of the subsequent financial market and thus make brown lenders believe that the holdout firm is likely to be the green type. In contrast to the public bailouts, green lenders do not necessarily suffer any financial loss because they take the anticipated high repayment term offered in the subsequent market as leverage to offer a significantly high repayment term under which the brown firm chooses the green project *ex-post*.

We found that sequential bidding for lending in the capital market is crucial to obtaining ESG integration. A natural follow-up question is what will occur when the green lenders are no longer the first movers. Theorem 3-(ii) argues that the welfare of green lenders may not be improved once green lenders lose their first-mover advantage. Recall from Lemma 1 that the green investment can be obtained only if the lenders can offer a contract with sufficiently large financial repayment terms. However, brown lenders in the early market will compete to finance the firm only for the sake of the financial payoff, bringing the firm's borrowing cost down to the levels that induce the brown investment. There is no effective way for green lenders to keep the firm's borrowing cost high when they cannot bid for funding earlier than brown lenders. Rather, the firm, whether it is the green or brown type, would rather borrow cheaply from brown lenders and leave the market early before receiving offers from green lenders in the subsequent capital market.

Our analysis suggests that granting green lenders a competitive advantage improves the quality of matching between different types of lenders and firms with regard to the facilitation of ESG integration. Particularly, the social benefit from the lender-borrower matching will be substantially improved when green lenders are matched with brown firms: the green lenders, strongly considering the social consequence of their funding, can influence the brown firms' investments in the green project by directly financing them. By contrast, the green firms with strong innate preferences for social value will invest in the green project no matter which type of lender is the source of funding (Lemma 3-(ii)).

Note that there will be no room for improvement in the matching quality in the capital market if all lenders have a homogeneous preference for social value: the borrowing firms, regardless of their own preferences for social value, must be matched with the lenders with the same characteristics. In Appendix D in the Online Appendix, we consider the market with green lenders only and prove that the equilibrium outcome remains unchanged whether some of the green lenders are granted a Stackelberg competitive advantage or all of them bid simultaneously in Bertrand fashion.

4.2 Policy Implications

Theorem 3 suggests important policy implications to achieve ESG integration in the presence of adverse selection on the firm's inherent preference for ESG. First, our analysis suggests that policymakers should consider the value-add of the incumbent ESG-friendly lenders in inducing borrowers to integrate ESG. Our analytical result implies that incumbent ESG-friendly lenders can "cleanse" the ESG market by taking out the brown firms. Thus, imposing a comparative disadvantage on ESG-friendly lenders may deter borrowing firms' ESG integration.

Indeed, ESG-endorsing lenders contributed to establishing a new capital market for ESG (e.g., the PRI that has existed since 2006) and led the efforts engaging with companies on ESG integration (Ceccarelli et al., 2021).[17] Further, these established ESG-endorsing lenders

[17] For example, there are several asset owners that committed to ESG integration, such as the Japanese GPIF ($1.8 trillion), the Norwegian Sovereign Wealth Fund ($1.3 trillion), the Korea National Pension Fund ($766 billion), the California Public Employees' Retirement System (CalPERS) ($426 billion), and the California State Teachers Retirement System (CalSTRS) ($259 billion) (go to https://www.pionline.com/interactive/worlds-largest-retirement-funds-2021 for more details). The total size of these funds is sizable compared to the entire US market capitalization that is estimated at $40 trillion in 2022 (go to https://siblisresearch.com/data/us-stock-market-value/ for more details). These owners already re-

are major players in the capital market.[18] For instance, the Japanese GPIF has allocated to asset managers and companies integrating ESG, and sought them to justify their ESG investment thesis.

Given this information, it is also conceivable that many corporate issuers have been borrowing from these incumbent large-sized institutional lenders to finance their ESG projects. Furthermore, when these firms seek capital to fund their new ESG projects, they are likely to engage the same lenders for financing the new projects again due to their existing relationship. If so, there is indeed a high likelihood that incumbent ESG-endorsing lenders would offer contracts to firms making new ESG investments earlier than other competing lenders that are new to the market.

Next, our paper also highlights the necessity for the regulators to consider ways in which asset allocators (e.g., pension funds and retail lenders) can evaluate the asset managers' willingness to genuinely integrate ESG. If such infrastructure is established, more capital will be effectively allocated to ESG-friendly managers, enhancing their market power in the ESG market and their ability to cleanse the ESG market. We view this implication as similar to (but a substantially developed version of) the SEC's recent policy proposal of the "naming rule" that permits asset managers to add the term "ESG" to the names of their mutual funds only when these managers are verified to have long committed to ESG.[19] Such regulation may be primarily designed to prevent asset managers from deceiving lenders in the ESG market. Nonetheless, ESG-friendly asset managers can also signal their ESG commitment by running mutual funds with names including the term "ESG" and play a role in "cleansing" the ESG market.

Lastly, our analysis provides an alternative explanation of the empirical observations deemed as an act of greenwashing by ESG-friendly lenders. For example, Kim and Yoon (2023) points out that the PRI signatories are not exhibiting ESG follow-through in the following 6 - 12 quarters after signing. Our main results from Theorem 3 suggest that these signatories (if they are truly willing to integrate ESG) are indeed cleansing the ESG market through two channels. First, the holdout firms who decide not to borrow from the PRI signatories can be perceived to have the willingness to integrate ESG by the other lenders, which addresses

quire asset managers to integrate the ESG metrics into their investment process and engage with their portfolio companies on critical ESG issues of their choosing (Henderson et al., 2019). Confirming this notion, Kim and Yoon (2023) find that the PRI signatories attracted significantly larger fund inflows from asset allocators after committing to incorporate ESG into their investment decisions.

[18]See https://www.pionline.com/interactive/worlds-largest-retirement-funds-2021 for details.

[19]See https://www.sec.gov/news/press-release/2022-91 for details.

the adverse selection problem in the ESG market. Next, the firms that borrow from the PRI signatories are incentivized to integrate ESG after accepting a high borrowing rate, even though they may not be strongly willing to integrate ESG.

5 Related Literature

We acknowledge papers that examine the ESG commitments and follow-through of issuers and investors. For example, Gibson Brandon et al. (2022) and Pastor, Stambaugh and Taylor (2023) document that some PRI signatories, especially those outside of the US, exhibit a moderate incorporation of ESG. On the other hand, Kim and Yoon (2023) and Liang, Sun and Teo (2022) examine funds that signed the PRI initiative and find that they exhibit little follow through on ESG while exhibiting lower alpha but attracting greater fund flows. On the issuer side, Raghunandan and Rajgopal (2022*b*) analyzes the Business Roundtable (BRT) firms and document no evidence that the signatories significantly engaged in stakeholder-centric practices. This also reflects the difficulty in disentangling actual ESG performance of companies from the information disclosed to outsiders and underscores an important policy issue on how much the governments, companies, and investors can or should act to deter greenwashing.

To this end, a stream of papers (Choy et al., 2021; Fiechter, Hitz and Lehmann, 2022; Christensen et al., 2017; Krueger et al., 2021) highlights the necessity of regulatory enforcement. On the other hand, Aghamolla and An (2023) and Xue (2023) argue that firms may inefficiently invest in ESG projects *ex ante* when the disclosure is mandatory or exceptionally precise due to the fear of being penalized for their poor ESG performance by outside lenders *ex post*. Relatedly, there are also papers that hint at the potential of market-driven solutions for the greenwashing problem, largely through voluntary disclosure (Rouen, Sachdeva and Yoon, 2022; Bochkay, Choi and Hales, 2022).

There is a growing theory literature that analyzes how the changes in lenders' attitudes towards ESG can influence socially responsible investments. Pástor, Stambaugh and Taylor (2021) finds that dispersion in lenders' concern with ESG is a necessary condition for the investment in ESG projects; green firms can enjoy a substantively low cost of capital only when the lenders' desire to pursue ESG even by sacrificing financial returns is not fully adjusted in financial markets. In contrast, Gupta, Kopytov and Starmans (2022) argues that firm managers may strategically delay their ESG integration decision until they meet an unexpected surge of lenders' demand for ESG and thus finance their ESG projects cheaply.

Another strand of literature focuses on crowd-out effects on socially responsible investments in asset markets. Green and Roth (2021) argues that it may be suboptimal for each individual ESG-friendly lender to maximize ESG values in her own portfolio instead of the aggregate social surplus. Regarding non-ESG lenders pursuing financial returns only, such an investment strategy crowds out firms that are not highly financially profitable but could generate positive (but relatively small) social returns, resulting in low aggregate social surplus. In addition, Bisceglia, Piccolo and Schneemeier (2023) suggests that ESG-friendly lenders should concentrate their resources on a selected group of firms to maximize the social impacts each targeted firm can generate. However, the concentration of capital also creates negative crowd-out effects: the excluded firms will be deprived of capital for ESG integration; only the selected firms will survive and thus obtain strong market power in a real economy, incurring an additional aggregate welfare loss.

Furthermore, there are papers that analyze how ESG-friendly lenders can incentivize borrowing firms to integrate ESG into their business. Heinkel, Kraus and Zechner (2001) argues that ESG-friendly lenders can influence borrowing firms' behavior by divesting the stocks of these firms. Hart and Zingales (2017) instead suggest an alternative "voice" strategy: ESG-friendly lenders should become major shareholders and thus directly influence firm managers' behavior by voting for ESG-endorsing corporate policies in shareholders' meetings. Other papers further analyze the effectiveness of the "voice" strategy in various firm ownership structures. Broccardo, Hart and Zingales (2022) argues that the firm ownership must be sufficiently diversified among shareholders, which substantially lowers the cost of ESG integration each individual shareholder shoulders. Edmans, Levit and Schneemeier (2022) argues that an ESG-friendly lender can reward a firm integrating ESG by purchasing a large number of shares and that such a practice can be more effective if firm managers are short-term oriented because the managers will be instantly compensated for raising the market value of shares. These papers assume that lenders are armed with the capability to reward or penalize borrowers' ex-post actions on ESG.

Lastly, recent papers focus on the optimal design of measuring and disclosing corporate borrowers' ESG activities in the presence of asymmetric information. Friedman, Heinle and Luneva (2021) shows that the optimal design of ESG reporting standards to tackle the greenwashing problem depends on the outside lenders' aggregate preferences for ESG. Bonham and Riggs-Cragun (2022) compares the impacts of different schemes on ESG (e.g., executive compensation, tax benefits, and regulations on disclosing firms' ESG activities. They identify the respective institution and market conditions for each of those schemes to effectively motivate regulated firms to integrate ESG.

Our research is most closely related to Oehmke and Opp (2024), which shows that ESG-friendly lenders can achieve ESG investments by attaching an ESG investment covenant to the terms of lending but sacrificing their financial payoffs. However, given the institutional frictions (e.g., the hard-to-verify-and-assure nature of firm ESG efforts), borrowers can deviate from their promises on ESG integration, which is the main focus of our current research. Our paper complements and extends the aforementioned literature by discovering the competition policies in ESG credit markets as a key solution for greenwashing problems before the establishment of a precise standard of monitoring corporate borrowers' ESG efforts and performances. Particularly, competitive advantages held by ESG-friendly lenders improve rather than hamper ESG integration, which has no analog in any other studies.

6 Conclusion

This paper presents a model of ESG integration where borrowers' promises on ESG are not contractible. Competing with non-ESG lenders considering financial payouts only, ESG-friendly lenders face difficulties ensuring that the borrowing firm integrates ESG. A key constraint on ESG integration is that it is practically impossible to prevent borrowers from deviating from their ESG promises and engaging in greenwashing. Before suggesting a relevant solution for ESG integration, the first essential work should be to understand the impact of the lack of contractual enforcement on the borrowers' incentive for ESG integration.

Our main findings are summarized as follows. Corporate issuers have the incentive to integrate ESG only when the cost of borrowing is high. Indeed, a high borrowing cost lowers private net financial gains of corporate issuers from not following through on their ESG promises and pursuing short-run financial returns. However, ESG-friendly lenders alone cannot achieve ESG integration due to the competition with non-ESG lenders — who only appreciate financial payoffs and thus are willing to finance cheaply once the borrowers undertake any financial project with a high financial return. Nevertheless, ESG-friendly lenders play a crucial role in achieving ESG integration in the presence of adverse selection on the corporate issuers' innate preferences for ESG. Particularly, ESG-friendly lenders with a first-mover advantage can finance and thus take out the firms with weak preferences for ESG early on from the financial market, creating the following positive effects. First, the firms not funded by the first-moving ESG-friendly lenders are perceived as those with strong preferences for ESG. Hence, the subsequent lenders require a high borrowing rate because of a low financial return from integrating ESG. Knowing this, the first-moving ESG-friendly lenders also charge a high borrowing rate to firms with weak preferences for ESG. These non-ESG firms then have no other option but to borrow at a high borrowing rate that incentivizes ESG integration.

Our research further suggests an important policy implication: promoting competition between lenders may not always facilitate ESG integration because corporate leaders can have a twofold incentive to deviate from ESG promises. First, a competitive lending market lowers the cost of borrowing, incentivizing firm managers to pursue short-run financial income instead of long-run social value. Second, the incumbent ESG-friendly lenders cannot "cleanse" the ESG market by taking out the firms that are not genuine about integrating ESG.

The central lesson from our research is that, with the lack of commitment, the impacts of funding conditions for borrowers and competition structures for lenders on ESG integration are completely different from the model allowing contractual enforcement. To the best of our knowledge, our work's motivations, insights, and policy implications are novel and have not been studied in the previous literature. We believe our research will open a new avenue for important policy debates and inform empirical works on greenwashing.

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Appendix

A Proofs Omitted in Section 3

Recall that we invoke two tie-breaking rules.

(T1) The firm breaks a tie by investing in the brown project whenever it finds the green and brown projects indifferent.

(T2) Every lender, regardless of her type, breaks a tie by offering a contract that yields a higher probability of funding whenever she is indifferent between two offers.

The firm always accepts the lowest repayment term among all available offers (Lemma 2). Furthermore, both the green and brown projects have a non-negative NPV (Assumption 1), and thus, the firm must be financed with probability one in equilibrium. These two observations, combined with (T1), imply that there are only two kinds of equilibrium: one in which the firm chooses the green project with probability one (referred to as *green equilibrium* hereafter), and another in which the firm chooses the brown project with probability one (referred to as *brown equilibrium* hereafter). In particular, the firm never randomizes its project selection on the path.

Let D^* generically refer to the lowest repayment term offered by the lenders in equilibrium. D^* is the offer that the firm eventually accepts. Hence, D^* will be referred to as the *equilibrium repayment term* or *winning offer* interchangeably. We will call the lenders who offer D^* in equilibrium *winning lenders*.

A.1 Proof of Theorem 1

Brown Equilibrium: We first prove that

$$D^* = \frac{1}{p_B} \leq \overline{D}(\lambda) \tag{A.1}$$

is the necessary and sufficient condition for the existence of a brown equilibrium. To prove the necessity, note first that $D^* \leq \overline{D}(\lambda)$ in any brown equilibrium (see Lemma 1). Furthermore, if any lender undercuts D^* by offering $D' < D^* \leq \overline{D}(\lambda)$, the firm would still choose the brown project. Thus, the standard argument in the Bertrand competition leads to $D^* = 1/p_B$, and all lenders earn zero profit in any brown equilibrium. This completes the proof of the necessity of (A.1).

To prove the sufficiency of (A.1), suppose $1/p_B \leq \overline{D}(\lambda)$ holds and then consider the case where all brown and green lenders offer $D^* = 1/p_B$ and the firm chooses the brown project. It is straightforward that no player has an incentive to deviate. In particular, the green lenders cannot alter the firm's project selection by undercutting D^*. This completes the proof of the sufficiency of (A.1).

Green Equilibrium: Next, we prove that

$$\overline{D}(\lambda) < \frac{1}{p_B} < D^* = \frac{1}{p_G} \tag{A.2}$$

is the necessary and sufficient condition for the existence of a green equilibrium. First, $D^* > \overline{D}(\lambda)$ in any green equilibrium (Lemma 1). Furthermore, the firm would still choose the green project if any lender deviates by offering $D' \in (\overline{D}(\lambda), R]$. Thus, by the standard argument in the Bertrand competition, $D^* = 1/p_G$ and all lenders earn zero profit in any green equilibrium. Finally, we also must have $\overline{D}(\lambda) < 1/p_B$. Otherwise, any brown lender could increase the probability of lending by deviating to $D = 1/p_B \leq \overline{D}(\lambda) < D^* = 1/p_G$, which contradicts the tie-breaking rule (T2). Note that such a deviation would induce the firm to choose the brown project (because $\overline{D}(\lambda) \geq D$), and thus, incur no financial loss for the deviating brown lender. This completes the proof of the necessity of (A.2).

To prove the sufficiency, suppose (A.2) holds and then consider the case that all brown and green lenders offer $D^* = 1/p_G$ and the firm chooses the green project. It is straightforward that no player has an incentive to deviate. In particular, no lender can benefit from undercutting D^*, as it would always yield the deviating lender a strictly negative financial payoff. This completes the proof of the sufficiency of (A.2).

A.2 Proof of Theorem 2

Brown Equilibrium: The condition $1/p_B \leq \overline{D}(\lambda)$ is the necessary and sufficient for the existence of a brown equilibrium. The proof is essentially identical to the case with both brown and green lenders in the market (Theorem 1), hence omitted here.

Green Equilibrium: Next, we prove that

$$\overline{D}(\lambda) < \min\left\{\frac{1+\phi}{p_B}, \frac{1}{p_G}\right\} \tag{A.3}$$

is the necessary and sufficient condition for the existence of a green equilibrium. First of all, $D^* > \overline{D}(\lambda)$ in any green equilibrium (Lemma 1). Furthermore, if any lender deviates by offering $D' \in (\overline{D}(\lambda), R]$, the firm would still choose the green project. Thus, the standard argument in the Bertrand competition applies to have $D^* = 1/p_G$. Furthermore, all green lenders obtain ϕ as final payoff in any green equilibrium. Finally, we also must have $\overline{D}(\lambda) < (1+\phi)/p_B$. Otherwise, a green lender could increase the probability of lending without incurring any loss in payoff by deviating to $D = (1+\phi)/p_B \leq \overline{D}(\lambda)$, contradicting the tie-breaking rule (T2) stated at the beginning of Appendix A. Note that such a deviation would induce the firm to choose the brown project (because $\overline{D}(\lambda) \geq D$), and thus, the deviating green lender still obtains ϕ as the final payoff. This completes the proof of the necessity of (A.3).

To prove the sufficiency, suppose (A.3) holds and then consider the case that all green lenders offer $D = D^* = 1/p_G$. Note that all green lenders earn ϕ as their expected final payoff in this case. It is straightforward that no lender has an incentive to deviate. In particular, no green lender can benefit from offering a repayment lower than D^*, as it would always yield the deviating lender a payoff strictly less than ϕ. This completes the proof of the sufficiency of (A.2).

Finally, we complete the proof by showing

$$\min\left\{\frac{1+\phi}{p_B}, \frac{1}{p_G}\right\} = \frac{1}{p_G}.$$

Note that

$$\frac{1}{p_G} < \frac{1+\phi}{p_B} \quad\Longleftrightarrow\quad p_B < p_G + p_G\phi \quad\Longleftrightarrow\quad \frac{p_B - p_G}{p_G} < \phi.$$

By Assumption 1-(ii), we have $R \geq 1/p_G$. In addition, we have $\phi > R(p_B - p_G)$ by Assumption 2. Combining these two inequalities, we obtain $\phi > (p_B - p_G)/p_G$.

B Proofs Omitted in Section 4

B.1 Preliminary Observations

We will discuss two versions of the lending game in this section: (i) a version in which all lenders make offers simultaneously (referred to as the static lending game hereafter) and (ii) a version in which some lenders offer in the early market while all other lenders offer in the later market (referred to as the dynamic lending game hereafter). In both static and dynamic versions of the lending game, we will focus on pure-strategy perfect Bayesian equilibrium with the tie-breaking rules (T1) and (T2) stated at the beginning of Appendix A being imposed.

In discussions on a dynamic version of the lending game, we will refer to the early market as the *first period* and the later market as the *second period*, respectively. Let n_G^1 and n_G^2 respectively denote the numbers of green lenders who offer in the first and second periods; similarly, let n_B^1 and n_B^2 denote the numbers of brown lenders who offer in the first and second periods, respectively. Finally, let $n_G = n_G^1 + n_G^2$ and $n_B = n_B^1 + n_B^2$ denote the total numbers of the respective types of lenders.

Let D^* generically refer to the lowest repayment term offered by the lenders in any equilibrium of the static lending game. D^* is the offer that the firm eventually accepts. Hence, we will call the lenders who offer D^* in equilibrium the *winning lenders*. Similarly, when we discuss a dynamic version of the lending game, let D_1^* and D_2^* refer to the lowest repayment term offers in the first and second periods, respectively, and $D^* \equiv \min\{D_1^*, D_2^*\}$. Note that our tie-breaking rules impose no restriction on the firm's decision between D_1^* and D_2^* in case $D_1^* = D_2^*$.

Finally, let q_2^* generically denote the probability that lenders in the second period of the dynamic lending game assign to the event $\lambda = \lambda_G$. Also, let $t_G \in [0,1]$ and $t_B \in [0,1]$ respectively refer to the probabilities that the green and brown firms accept D_1^* in equilibrium. By Bayes' rule,

$$q_2^* = \frac{(1-t_G)q}{(1-t_G)q + (1-t_B)(1-q)} \tag{B.1}$$

in any equilibrium such that $q(1-t_G) + (1-q)(1-t_B) > 0$ (i.e., in any equilibrium in which the firm rejects D_1^* with positive probability).

We first prove two preliminary lemmas. For these two lemmas (Lemma B.1 and B.2), we allow the prior belief q to be zero or one, whereas we assume $q \in (0,1)$ in the main text. Also, note that we do not impose Assumption 3 in Lemma B.1.

Lemma B.1. *Suppose that Assumptions 1 and 2 hold. Also, suppose that $q \in [0,1]$. All lenders earn zero financial profit in any equilibrium of the static lending game with more than two lenders (i.e., $n_B + n_G \geq 2$).*

Proof. Suppose for contradiction that there is an equilibrium in which winning lenders earn non-zero expected financial profit. There are three subcases. First, consider the case $D^* \notin \{\overline{D}(\lambda_B), \overline{D}(\lambda_G)\}$. For any sufficient small $\epsilon > 0$, all repayment terms $(D^* - \epsilon, D^* + \epsilon)$ induce the same project selection *ex post* if accepted by the firm (Lemma 1). Thus, by the standard argument in the Bertrand competition, all lenders must earn zero financial profit in equilibrium.

Next, suppose $D^* = \overline{D}(\lambda_G)$. Both firm types would choose the brown project in this case (Lemma 1), and thus, no social value is generated. To avoid the impossible case where the winning lenders earn a negative equilibrium payoff, we may assume $1/p_B < D^* = \overline{D}(\lambda_G)$ so that the winning lenders earn strictly positive financial profits in equilibrium. Note that the firm eventually makes a contract with only one lender. Hence, there must be lenders whose offers are rejected with a positive probability. By a standard Bertrand undercutting argument, these lenders would benefit if they undercut D^*, contradicting the presupposition that D^* is the equilibrium repayment term.

Finally, consider the case $D^* = \overline{D}(\lambda_B) > \overline{D}(\lambda_G)$, and thus, the green type of the firm chooses the green project, and the brown-typed firm chooses the brown project, respectively. The lender who eventually makes a contract with the firm will earn $[qp_G + (1-q)p_B]\overline{D}(\lambda_B) - 1$ as final financial profit. If this financial profit is strictly positive, other lenders could benefit by undercutting D^*. Thus, assume without loss

$$[qp_G + (1-q)p_B]\overline{D}(\lambda_B) - 1 < 0 \quad \Longrightarrow \quad \overline{D}(\lambda_B) < \frac{1}{qp_G + (1-q)p_B} \leq \frac{1}{p_G} \tag{B.2}$$

The brown lenders do not value the social value of the green project. Thus, all winning lenders must be green, in which case her equilibrium expected payoff is bounded from above by

$$\overline{\pi}_G := \frac{[qp_G + (1-q)p_B]\overline{D}(\lambda_B) - 1}{N^*} + q\phi < q\phi \tag{B.3}$$

where $q\phi$ captures the expected social value $q\phi$ this lender enjoys in equilibrium, and N^* is the number of the winning lenders.

Now, suppose that one of the winning green lenders deviates to $D' \in (D^*, 1/p_G) =$

$(\overline{D}(\lambda_B), 1/p_G)$. If there are other lenders who still offer D^* (i.e., if $N^* \geq 2$), the deviating lender strictly benefits from offering D' because it only saves their financial loss without changing the firm's project selection. If there is no other winning lender (i.e., if $N^* = 1$), both types of the firm would accept D' and then choose the green project. As a result, the deviating green lender would obtain a final payoff of $p_G D' - 1 + \phi$, where

$$\begin{aligned} p_G D' - 1 + \phi &> p_G \overline{D}(\lambda_B) - 1 + \phi \\ &= [qp_G + (1-q)p_B]\overline{D}(\lambda_B) - 1 + q\phi + (1-q)[\phi - (p_B - p_G)\overline{D}(\lambda_B)] \\ &> \overline{\pi}_G + (1-q)\left[\phi - \tfrac{p_B - p_G}{p_G}\right] \\ &> \overline{\pi}_G, \end{aligned}$$

where the second inequality follows from (B.2) and (B.3), and the last inequality follows from Assumption 1 and 2 (see the end of Section A.2). In sum, the deviating green lender would be better off in all cases, contradicting the presupposition that $D^* = \overline{D}(\lambda_B)$ is the equilibrium repayment term. *Q.E.D.*

Lemma B.2. *Suppose that Assumptions 1-3 hold. Also, suppose that $q \in [0, 1]$. Then, $D^* = 1/[qp_G + (1-q)p_B]$ in any equilibrium of the static lending game with more than two lenders, where at least one of them is brown (i.e., $n_B + n_G \geq 2$ and $n_B \geq 1$). In this type of equilibrium, only the green firm chooses the green project, whereas the brown firm chooses the brown project.*

Proof. All lenders must earn zero expected financial profit in equilibrium (Lemma B.1), which could be the case under Assumption 3-(i) only in the following two cases:

$$D^* = \frac{1}{qp_G + (1-q)p_B} \quad \text{or} \quad D^* = \frac{1}{p_G}$$

Note that $1/[qp_G + (1-q)p_B] \in [\overline{D}(\lambda_B), \overline{D}(\lambda_G))$ and $1/p_G^* > \overline{D}(\lambda_B)$ by Assumption 3. Thus, the brown firm chooses the brown project in the former case ($D^* = 1/[qp_G + (1-q)p_B]$) and the green project in the latter case ($D^* = 1/p_G$); the green firm chooses the green project in both cases.

Next, we show that there is no equilibrium such that $D^* = 1/p_G$; in other words, $D^* = 1/[qp_G + (1-q)p_B]$ in all equilibria. Suppose for contradiction there is an equilibrium such that $D^* = 1/p_G$. Pick a brown lender, whose equilibrium payoff must be zero by Lemma B.1. If this lender deviates to $D' \in (\overline{D}(\lambda_B), D^*)$, the firm would accept D' and then choose the brown and green projects with probabilities $1-q$ (if $\lambda = \lambda_B$) and q (if $\lambda = \lambda_G$), respectively. Thus, the deviation yields the brown project a payoff

$$[qp_G + (1-q)p_B]D' - 1 > [qp_G + (1-q)p_B]\overline{D}(\lambda_B) - 1 \geq 0$$

where the last weak inequality follows from Assumption 3-(ii). This contradicts the presupposition that $D^* = 1/p_G$ is the equilibrium repayment term. *Q.E.D.*

B.2 Proof of Lemma 3

We first prove Lemma 3-(ii) and then Lemma 3-(i). In the main text, we assume that only (a subset of) green lenders offer in the first period, and all brown lenders offer in the second period. However, Lemma 3-(i) holds for a more general case. The proof below is valid as long as at least one lender offers in the first period and multiple lenders offer in the second period (i.e., as long as $n_1^B + n_1^G \geq 1$ and $n_2^B + n_2^G \geq 2$ hold) regardless of their types.

B.2.1 Proof of Lemma 3-(ii)

First of all, we must have $D_2^* > \overline{D}(\lambda_G)$ in any equilibrium. This is an immediate consequence of Lemma B.1: all lenders' financial profit could be zero in the second period only if $D_2^* > \overline{D}(\lambda_G)$ because, under Assumption 3-(i), $\overline{D}(\lambda_G)$ is less than both $1/p_B$ and $1/p_G$.

To prove that the green-type firm always chooses the green project, suppose to the contrary that there is an equilibrium in which the green-type firm chooses the brown project with a positive probability. By Lemma 1, such an equilibrium can be supported only if $D^* = \min\{D_1^*, D_2^*\} \leq \overline{D}(\lambda_G)$. Since $D_2^* > \overline{D}(\lambda_G)$ as we already proved in the previous paragraph, we must have $D^* = D_1^* \leq \overline{D}(\lambda_G) < D_2^*$. Hence, both green and brown firms will accept D_1^* in the first period, and then, choose the brown project (Lemma 1). This implies that any lender from whom the firm accepts D_1^* obtain a negative financial profit equal to

$$p_B D^* - 1 \leq p_B \overline{D}(\lambda_G) - 1 < 0,$$

where the last strict inequality is due to Assumption 3-(i). However, this lender could get a strictly higher payoff (zero payoff) by withdrawing their offer D^*, contradicting the presupposition that D^* is the equilibrium repayment term.

B.2.2 Proof of Lemma 3-(i)

Suppose, for contradiction, that there is an equilibrium such that $q_2^* < q$, where q_2^* refers to the posterior probability that lenders in the second period assign to the event $\lambda = \lambda_G$. This is the case only if $D_1^* \le D_2^*$ and the firm accepts D_1^* in the first period with positive probability (i.e., $qt_G + (1-q)t_B > 0$); if both types reject D_1^*, q_2^* necessarily equals q. Also, from (B.1), we have

$$q_2^* = \frac{(1-t_G)q}{(1-t_G)q + (1-t_B)(1-q)} > q \iff t_G < t_B, \tag{B.4}$$

hence we may also assume $t_G \ge t_B$ without loss. Finally, applying Lemma B.2 to the later market with posterior $q_2^* < q$,

$$\overline{D}(\lambda_G) < D_1^* \le D_2^* = \frac{1}{q_2^* p_G + (1-q_2^*)p_B} < \frac{1}{qp_G + (1-q)p_B} \le \overline{D}(\lambda_B), \tag{B.5}$$

where the first inequality $\overline{D}(\lambda_G) < D_1^*$ follows from Lemma 1 and Lemma 3-(ii); the last inequality $1/(qp_G + (1-q)p_B) \le \overline{D}(\lambda_B)$ holds due to Assumption 3-(ii). Hence, once accepted by the firm, both D_1^* and D_2^* induce the green firm to choose the green project and the brown firm to choose the brown project.

We will complete the proof by showing that lenders who offer D_1^* have an incentive to deviate. We focus on the case where there is only one lender who offers D_1^*; the analysis of the case where multiple lenders offer D_1^* is similar hence we omit it here. We first show that the lender who offers D_1^* earns a negative financial profit. To see this, note that the firm would choose the green project with probability $qt_G/[qt_G + (1-q)t_B]$ and the brown project with probability $(1-q)t_B/[qt_G + (1-q)t_B]$ after it accepts D_1^*. Thus, conditional on the event that the firm accepts D_1^*, the lender's expected financial profit, which will we denote by L^*, is

$$\begin{aligned} L^* := \underbrace{\left[\frac{qt_G p_G + (1-q)t_B p_B}{qt_G + (1-q)t_B}\right]}_{\text{increasing in } t_B} D_1^* - 1 &\le \underbrace{\left[\frac{qt_G p_G + (1-q)t_G p_B}{qt_G + (1-q)t_G}\right]}_{qp_G + (1-q)p_B} D_1^* - 1 \qquad \because t_G \ge t_B \\ &< \frac{qp_G + (1-q)p_B}{qp_G + (1-q)p_B} - 1 = 0 \qquad \because \text{(B.5)}. \end{aligned}$$

Now, suppose that this lender deviates and withdraws their offer D_1^*. Then, both types of the firm will accept D_2^* in the second period. Recall that D_1^* and D_2^* induce the same project selection. Thus, this deviation only saves the lender's financial loss L^* without changing the firm's project selection; hence, the lender can strictly benefit from the deviation.

B.3 Proof of Theorem 3

B.3.1 Proof of Theorem 3-(i)

We first show that the posterior belief q_2^* needs to be strictly larger than the prior belief q in any equilibrium such that the brown firm chooses the green project. To see this, note first that the brown firm would choose the green project only if $D^* = \min\{D_1^*, D_2^*\} > \overline{D}(\lambda_B)$ (Lemma 1). If the posterior belief q_2^* is weakly less than q, however, we have

$$\overline{D}(\lambda_B) \geq \frac{1}{qp_G + (1-q)p_B} \geq \frac{1}{q_2^* p_G + (1-q_2^*)p_B}$$

and the lenders in the later market will offer $D_1^* = 1/[q_2^* p_G + (1-q_2^*)p_B] \leq \overline{D}(\lambda_B)$ (see Lemma B.2). This completes the proof of the necessity of $q < q_2^*$ to induce the brown firm to choose the green project.

Note that, by Lemma 1, there is no randomness in either firm type's final project selection on the path of any equilibrium; there is still randomness in from whom and in which period the firm accepts D^*. Furthermore, by Lemma 3-(ii), the green firm always chooses the green project in any equilibrium. To prove Theorem 3-(i), therefore, it suffices to construct two equilibria: one in which the brown firm chooses the brown project and another in which it chooses the green project.

We first construct an equilibrium strategy profile such that the brown firm chooses the brown project as follows. Lemma 2 uniquely pins down each firm type's project selection after accepting any repayment term both on and off the path. Thus, we will omit the firm's equilibrium project selection when we describe the firm's equilibrium strategy.

- All lenders (both green and brown lenders) in both periods offer $D^* = 1/[qp_G + (1-q)p_B]$.
- Let $\underline{D}_1$ denote the lowest repayment term offered in the first period ($\underline{D}_1 = D^*$ on the path). The firm accepts $\underline{D}_1$ in the first period if and only if $\underline{D}_1 \leq D^*$.
- Let $\underline{D}_2$ denote the lowest repayment term offered in the second period. The firm accepts $\underline{D}_2$ in the second period if and only if $\underline{D}_2 \leq R$.
- The lenders in the later market hold the following posterior belief: $Pr(\lambda = \lambda_G) = q$.

Note that the firm accepts D^* in the first period and then chooses the green project (respectively, the brown project) if and only if $\lambda = \lambda_G$ (respectively, if and only if $\lambda = \lambda_B$).

It is straightforward to prove that no player has any incentive to deviate from the above strategy profile; hence, we omit its proof here. The second period is not reached on the equilibrium path. Thus, the posterior belief in the last bullet point is *consistent*, which in turn justifies the lenders' equilibrium offer D^* in the second period (Lemma B.2).

Next, we construct an equilibrium strategy profile such that both green and brown firms choose the green project. Again, we will omit the firm's equilibrium project selection when we describe the firm's equilibrium strategy. Choose two real numbers t_G and $t_B \in [0, 1]$ such that at least one of t_G and t_B is strictly less than 1 and

$$\frac{q(1-t_G)}{q(1-t_G)+(1-q)(1-t_B)} > \frac{p_B\overline{D}(\lambda_B)-1}{(p_B-p_G)\overline{D}(\lambda_B)} \tag{B.6}$$

where the right hand side of (B.6) lies in $[0, q] \subset [0, 1)$ by Assumption 3-(i). For example, $t_G = \epsilon$ and $t_B = 1-\epsilon$ satisfy (B.6) whenever $\epsilon \geq 0$ is sufficiently close to or exactly equal to 0. For any pair (t_G, t_B) that satisfies (B.6), consider the following strategy profile:

- All lenders (both green and brown lenders) in both periods offer $D^* = 1/p_G$.
- Let $\underline{D}_1$ denote the lowest repayment term offered in the first period ($\underline{D}_1 = D^*$ on the path). The firm of type $\lambda_k \in \{\lambda_B, \lambda_G\}$ accepts $\underline{D}_1$ in the first period with probability $x_k(\underline{D}_1)$, where
$$x_k(\underline{D}_1) = \begin{cases} 0 & \text{if } \underline{D}_1 > D^*, \\ t_k & \text{if } \underline{D}_1 = D^*, \\ 1 & \text{if } \underline{D}_1 < D^*. \end{cases}$$
- Let $\underline{D}_2$ denote the lowest repayment term offered in the second period. The firm accepts $\underline{D}_2$ in the second period if and only if $\underline{D}_2 \leq R$.
- The lenders in the second period form the following consistent posterior belief using the Bayes' rule:
$$Pr(\lambda = \lambda_G) = q_2^* = \frac{q(1-t_G)}{q(1-t_G)+(1-q)(1-t_B)}. \tag{B.7}$$

Note that $D^* = 1/p_G$ is larger than both $\overline{D}(\lambda_B)$ and $\overline{D}(\lambda_G)$. Hence, both firm types choose the green project on the path, yielding the respective equilibrium payoffs zero and ϕ to the brown and green lenders.

To see that no green lender in the first period has an incentive to deviate, suppose that a green lender in the first period undercuts $D^* = 1/p_G$ with $D' < D^*$. It is straightforward that the deviation would result in a negative financial profit whenever $D' \in [0, \overline{D}(\lambda_G)] \cup (\overline{D}(\lambda_B), D^*)$. Thus, let us focus on the case $D' \in (\overline{D}(\lambda_G), \overline{D}(\lambda_B)]$, which would induce the green firm to choose the green project and the brown firm to choose the brown project. Then, the deviating lender would obtain $[qp_G + (1-q)p_B]D' - 1 + q\phi$ as final payoff, where

$$[qp_G + (1-q)p_B]D' - 1 + q\phi < \frac{qp_G + (1-q)p_B}{p_G} - 1 + q\phi = (1-q)\frac{p_B - p_G}{p_G} + q\phi < \phi.$$

Here, the first inequality follows from $D' < D^* = 1/p_G$, and the second inequality follows from $\phi > (p_B - p_G)/p_G$ by Assumption 1 and 2 (see the end of Section A.2). Hence, the deviation always yields a payoff strictly lower than the equilibrium payoff ϕ.

Next, to show that lenders in the second period also have no incentive to deviate, suppose that any of them undercuts $D^* = 1/p_G$ by offering $D' < D^*$. Again, let us focus on the case $D' \in (\overline{D}(\lambda_G), \overline{D}(\lambda_B)]$. With the posterior belief (B.7), the deviating lender's expected financial profit from the deviation is negative:

$$\begin{aligned}
[p_G q_2^* + p_B(1-q_2^*)]D' - 1 &\leq \underbrace{[p_G q_2^* + p_B(1-q_2^*)]}_{\text{decreasing in } q_2^*}\overline{D}(\lambda_B) - 1 && \because D' \leq \overline{D}(\lambda_B) \\
&< \left[\frac{p_G(p_B\overline{D}(\lambda_B) - 1)}{(p_B - p_G)\overline{D}(\lambda_B)} + \frac{p_B(1 - p_G\overline{D}(\lambda_B))}{(p_B - p_G)\overline{D}(\lambda_B)}\right]\overline{D}(\lambda_B) - 1 && \because \text{(B.6)} \\
&= 0.
\end{aligned}$$

This proves that the deviation is strictly unprofitable. In sum, all lenders have no incentive to deviate from D^*.

B.3.2 Proof of Theorem 3-(ii)

We first consider the case where all green and brown lenders simultaneously offer loans, which corresponds to the static version of the lending game with $n_B \geq 2$ and $n_G \geq 2$. Lemma B.2 applies to this game; hence, we must have $D^* = 1/[qp_G + (1-q)p_B] \leq \overline{D}(\lambda_B)$ and the brown firm chooses the brown project in all equilibria.

Now, consider the case where at least one brown lender bids in the first period, together with other lenders. This case corresponds to the dynamic version of the lending game with $n_B^1 \geq 1$ and $n_B^1 + n_G^1 \geq 2$. Suppose, for contradiction, that there is an equilibrium such that both green and brown types of the firm choose the green project on the path. By Lemma 1 and 2, this is the case only if

$$D^* > \max\left\{\overline{D}(\lambda_B), \overline{D}(\lambda_G)\right\} = \overline{D}(\lambda_B).$$

Furthermore, to exclude the possibility of undercutting D^*, we also must have $D^* = 1/p_G$, and thus, all lenders earn zero financial profit on the path. Finally, by the tie-breaking rule (T2) stated at the beginning of Appendix A, all lenders in the first period must offer D^*, so that all of them can make a contract with the firm with positive probability (but still less than one).

Now, suppose that a brown lender in the first period deviates by offering $D' = \overline{D}(\lambda_B)$. Both types of the firm will accept D', and then the brown firm (respectively, the green firm) will choose the brown project (respectively, the green project). In the end, the deviation yields the brown lender

$$[qp_B + (1-q)p_G]\overline{D}(\lambda_B) - 1$$

as the final payoff, which is non-negative by Assumption 3-(ii). Thus, the brown lender can increase their probability of lending by offering $D' = \overline{D}(\lambda_B)$, without incurring any loss in final payoff. This contradicts the presupposition that D^* is the equilibrium repayment (and hence, no lender has an incentive to undercut D^*).

Online Appendix: Supplementary Analysis

We provide supplementary analysis of the model with adverse selection presented in Section 4. Throughout this section, we will adopt the definitions and notations introduced in Section B.1.

C The Capital Market with Brown and Green Lenders

In this section, we show that the same equilibrium outcome arises in both static and dynamic versions of the lending game whenever Assumption 3 fails. In both static and dynamic versions of the lending game, we will focus on perfect Bayesian equilibrium such that *(a)* all players play pure strategies, and *(b)* the tie-breaking rules (T1) and (T2) stated at the beginning of Appendix A are imposed.

We first focus on Assumption 3-(i). With Proposition C.1 – C.3 below combined, we will be able to conclude that, if Assumption 3-(i) fails, granting green lenders a competitive advantage *à la* Stackelberg cannot make any change to the equilibrium outcome.

Proposition C.1. *Suppose Assumption 1 and 2 hold. Additionally, suppose* $\overline{D}(\lambda_B) < 1/p_B$*.*

(i) Both brown and green types of the firm choose the green project in any equilibrium of the static version of the lending game with $n_G \geq 1$ *and* $n_B \geq 2$*.*

(ii) Both brown and green types of the firm choose the green project in any equilibrium of the dynamic version of the lending game with $n_G^1 \geq 1$ *and* $n_B^2 \geq 2$*.*

Proof. It is straightforward that, for both the static and dynamic versions of the lending game, there exists an equilibrium such that all lenders offer $D^* = 1/p_G$, and hence, both firm types choose the green project. In what follows, we will prove that there is no other equilibrium.

First, consider the static lending game. Note that all lenders earn zero financial profit in any equilibrium (Lemma B.1). With $\overline{D}(\lambda_B)$ being strictly less than both $1/p_G$ and $1/p_B$, the winning lenders earn zero financial profit only if $D^* = 1/p_G$. This proves that there is no other equilibrium for the static lending game.

Next, consider the dynamic lending game. Lemma B.1 still applies to the continuation game played in the second period. Thus, by an argument essentially identical to one in the last paragraph, we must have $D_2^* = 1/p_G$ in any equilibrium. Given this, the lenders in the first period also have no incentive to undercut $D^* = 1/p_G$ because any $D' < 1/p_G \leq \overline{D}(\lambda_G)$ would result in a negative financial profit and possibly less social value. *Q.E.D.*

From now on, we will restrict attention to the case $1/p_B \leq \overline{D}(\lambda_B)$. The next proposition considers the case where $1/p_B$ is even smaller than another cutoff, $\overline{D}(\lambda_G)$.

Proposition C.2. *Suppose Assumption 1 and 2 hold. Additionally, suppose $1/p_B \leq \overline{D}(\lambda_G)$.*

(i) Both brown and green types of the firm choose the brown project in any equilibrium of the static version of the lending game with $n_G \geq 1$ and $n_B \geq 2$.

(ii) Both brown and green types of the firm choose the brown project in any equilibrium of the dynamic version of the lending game with $n_G^1 \geq 1$ and $n_B^2 \geq 2$.

Proof. It is straightforward that, for both the static and dynamic versions of the lending game, there exists an equilibrium such that all lenders offer $D^* = 1/p_B$, and hence, both firm types choose the brown project. In what follows, we will prove that there is no other equilibrium.

First, consider the static lending game. Suppose for contradiction that there is an equilibrium with $D^* \neq 1/p_B$. We may focus on the case $D^* > 1/p_B$ because any repayment term lower than $1/p_B$ results in a negative payoff for a lender. Note that all lenders earn zero financial profit in any equilibrium (Lemma B.1). Furthermore, the firm eventually makes a contract with only one lender. Hence, there must be a brown lender such that (i) her equilibrium payoff is zero, and (ii) her offer is rejected by the firm with a positive probability. Pick one of those brown lenders and suppose that she deviates to $D' = 1/p_B < D^*$. Then, both types of the firm will accept D' and then choose the brown project (because $D' = 1/p_B \leq \overline{D}(\lambda_G) < \overline{D}(\lambda_B)$). Hence, the brown lender could increase her probability of lending without any loss in her financial payoff by the deviation. This contradicts the tie-breaking rule (T2).

Next, consider the dynamic lending game. Lemma B.1 applies to the continuation game played in the second period. Thus, by an argument essentially identical to the last paragraph, we must have $D_2^* = 1/p_B$ in any equilibrium. Given this, the lenders in the first period also have no incentive to undercut $D^* = 1/p_B$ because any $D' < 1/p_B \leq \overline{D}(\lambda_G)$ would induce both types to choose the brown project, and thus, result in a negative financial profit without any social value. *Q.E.D.*

The next proposition concerns the case where both $1/p_B$ and $1/p_G$ lie between $D(\lambda_G)$ and $\overline{D}(\lambda_B)$, and hence,

$$\overline{D}(\lambda_G) < \frac{1}{qp_G + (1-q)p_B} \leq \overline{D}(\lambda_B) \quad \forall q \in [0,1].$$

Proposition C.3. *Suppose Assumption 1 and 2 hold. Additionally, suppose* $\overline{D}(\lambda_G) < 1/p_B < 1/p_G \leq \overline{D}(\lambda_B)$.

(i) The brown firm (respectively, the green firm) chooses the brown project (respectively, the green project) in any equilibrium of the static version of the lending game with $n_G \geq 1$ *and* $n_B \geq 2$.

(ii) The brown firm (respectively, the green firm) chooses the brown project (respectively, the green project) in any equilibrium of the dynamic version of the lending game with $n_G^1 \geq 1$ *and* $n_B^2 \geq 2$.

Proof. It is straightforward that, for both the static and dynamic versions of the lending game, there exists an equilibrium such that all lenders offer $D^* = 1/[qp_G + (1-q)p_B] \in (\overline{D}(\lambda_G), \overline{D}(\lambda_B)]$, and hence, each respective firm type chooses a project as described in the proposition. In what follows, we will prove that there is no other kind of equilibrium.

First, consider the static lending game. Note that all lenders earn zero financial profit in any equilibrium (Lemma B.1). With $\overline{D}(\lambda_B)$ being strictly less than both $1/p_G$ and $1/p_B$, the winning lenders earn zero financial profit only if $D^* = 1/[qp_G + (1-q)p_B]$. This proves that there is no other equilibrium for the static lending game.

Next, consider the dynamic lending game. Lemma B.1 applies to the continuation game played in the second period. Thus, by an argument essentially identical to the last paragraph, we must have $D_2^* = 1/[q_2^* p_G + (1-q_2^*)p_B]$ in any equilibrium. Note that $D_2^* \in (\overline{D}(\lambda_G), \overline{D}(\lambda_B)]$ for any $q_2^* \in [0,1]$ since $\overline{D}(\lambda_G) < 1/p_B < 1/p_G \leq \overline{D}(\lambda_B)$. If $\overline{D}(\lambda_G) < D_2^* < D_1^*$, we have

$$D^* = \min\{D_1^*, D_2^*\} = D_2^* \in (\overline{D}(\lambda_G), \overline{D}(\lambda_B)].$$

We also have $D^* = \min\{D_1^*, D_2^*\} = D_1^* \in (\overline{D}(\lambda_G), \overline{D}(\lambda_B)]$ if $\overline{D}(\lambda_G) < D_1^* \leq D_2^*$. Hence, the firm's equilibrium project selection would coincide with the one in Proposition C.3-(ii). Thus, we may focus on the case $D_1^* \leq \overline{D}(\lambda_G) < D_2^*$ without loss. In this case, both types of the firm accept D_1^* and then choose the brown project. However, any lender from whom the firm accepts D_1^* would obtain a strictly negative payoff, which cannot occur in any equilibrium. *Q.E.D.*

Proposition C.4. *Suppose Assumption 1 and 2 hold. Additionally, suppose that Assumption 3-(i) holds but Assumption 3-(ii) fails.*

(i) Both brown and green types of the firm choose the green project in any equilibrium of the static version of the lending game with $n_G \geq 1$ and $n_B \geq 2$.

(ii) Both brown and green types of the firm choose the green project in any equilibrium of the dynamic version of the lending game with $n_G^1 \geq 1$ and $n_B^2 \geq 2$.

Proof. We can rewrite the assumption imposed in this proposition as follows:

$$\overline{D}(\lambda_G) < \frac{1}{p_B} \leq \overline{D}(\lambda_B) < \frac{1}{qp_G + (1-q)p_B} < \frac{1}{p_G}. \tag{C.1}$$

It is straightforward that, for both the static and dynamic versions of the lending game, there exists an equilibrium such that all lenders offer $D^* = 1/p_G$, and hence, both firm types choose the green project. In what follows, we will prove that there is no other equilibrium. This is clearly true for the static version of the lending game, in which all lenders earn zero financial profit in any equilibrium (Lemma B.1); with (C.1), the winning lenders earn zero financial profit only if $D^* = 1/p_G$.

In what follows, we focus on the dynamic lending game. Note first that the proof of Lemma 3-(ii) does not rely on Assumption 3-(ii). Thus, the green firm chooses the green project in any equilibrium, which allows us to assume without loss that $\overline{D}(\lambda_G) < D^*$. Suppose to the contrary that there is an equilibrium in which the brown firm chooses the brown project. By Lemma 1, the brown firm chooses the brown project only if $D^* \leq \overline{D}(\lambda_B)$. Combining (C.1) as well as all the inequalities we have established so far, we obtain

$$\overline{D}(\lambda_G) < D^* = \min\{D_1^*, D_2^*\} \leq \overline{D}(\lambda_B) < \frac{1}{qp_G + (1-q)p_B}. \tag{C.2}$$

Step 1. We first claim that all lenders (regardless of their types) must earn a non-negative financial profit in equilibrium. By Lemma B.1, all lenders in the second period always get non-negative financial payoffs, so it suffices to show that lenders in the first period also get non-negative financial payoffs. Suppose to the contrary that a lender in the first period earns a negative financial payoff in equilibrium. We will call this lender the *designated lender* hereafter. Brown lenders cannot bear any financial loss. Hence, the designated lender must be green.

We first make some preliminary observations. First, the designated lender makes a negative financial profit only if the firm accepts her repayment term; hence, we may assume $D^* = D_1^*$ and the firm accepts D_1^* with a positive probability. Second, the non-financial externalities $q\phi$ is generated in equilibrium. Thus, the equilibrium payoff of the designated lender is strictly less than $q\phi$. Lastly, let $\underline{D}'$ denote the lowest repayment term offer among all other lenders. We must have $D^* \leq \underline{D}'$ and hence $\overline{D}(\lambda_G) < \underline{D}'$.

Now, we claim that the designated lender could improve her payoff by deviating to a higher repayment term $D' > \underline{D}'$. Since the firm accepts the offer with the lowest repayment term (Lemma 2), such a deviating offer D' will give zero financial payoff to the designated lender. Furthermore, since $\overline{D}(\lambda_G) < \underline{D}'$, the green firm continues choosing the green project. Hence, the deviation to D' does not change the amount of the expected non-financial externalities. In sum, the designated lender could strictly improve her payoff by deviating from the equilibrium strategy. The observation so far contradicts the presupposition that all lenders (including the designated lender) have no incentive to deviate from their equilibrium bidding strategies. This proves that no lender earns a negative financial profit in equilibrium.

Step 2. Next, we show that it is impossible that all lenders earn non-negative financial payoffs simultaneously. Let $t_k \in [0, 1]$ denote the probability that the firm of type $\lambda_k \in \{\lambda_G, \lambda_B\}$ finances its project in the first period. There are three possible cases. First, consider the case $t_G = t_B = 1$. By (C.2), the green firm chooses the green project and the brown firm chooses the brown project, respectively, after they accept D^* in the first period. Hence, any lender who finances the firm in the first stage must get the following financial payoff:

$$[qp_G + (1-q)p_B]D^* - 1 \leq [qp_G + (1-q)p_B]\overline{D}(\lambda_B) - 1 < 0,$$

where the first inequality follows from (C.2) and the second inequality follows from (C.1). Similarly, if $t_B = t_G = 0$ (and hence, $q_2^* = q$), any lender in the second period who offers D_2^* must earn a negative financial payoff.

Finally, suppose $t_k \in (0, 1)$ for some $k \in \{B, G\}$. Such randomization is justified only if $D^* = D_1^* = D_2^*$. Then the financial payoff of a lender from whom the firm accepts D^* in the first period is

$$[p_G\tilde{q}_1 + p_B(1-\tilde{q}_1)]D^* - 1 \qquad \text{where} \qquad \tilde{q}_1 = \frac{qt_G}{qt_G + (1-q)t_B} \tag{C.3}$$

Here, $\tilde{q}_1$ is the probability that the firm borrowing from the lender is of type λ_G. On the other hand, a lender from whom the firm accepts D^* in the second period gets the following financial payoff:

$$[p_G\tilde{q}_2 + p_B(1-\tilde{q}_2)]D^* - 1, \qquad \text{where} \qquad \tilde{q}_2 = \frac{q(1-t_G)}{q(1-t_G)+(1-q)(1-t_B)}. \tag{C.4}$$

Here, $\tilde{q}_2$ is the probability that the firm borrowing from the lender is of type λ_G. Since the brown lender's financial payoff (C.4) must be zero by Lemma B.1, we must have

$$D^* = \frac{1}{p_G\tilde{q}_2 + p_B(1-\tilde{q}_2)}. \tag{C.5}$$

Substituting D^* in (C.2) with (C.5), we have

$$D^* = \frac{1}{p_G\tilde{q}_2 + p_B(1-\tilde{q}_2)} \leq \overline{D}(\lambda_B) < \frac{1}{qp_G + (1-q)p_B} \quad \Longleftrightarrow \quad q > \tilde{q}_2. \tag{C.6}$$

In addition, substituting D^* in (C.3) with (C.5), (C.3) is non-negative if and only if

$$\frac{p_G\tilde{q}_1 + p_B(1-\tilde{q}_1)}{p_G\tilde{q}_2 + p_B(1-\tilde{q}_2)} \geq 1 \quad \Longleftrightarrow \quad \tilde{q}_1 \leq \tilde{q}_2 \quad \Longleftrightarrow \quad t_G \leq t_B.$$

But the inequality $t_G \leq t_B$ implies

$$\tilde{q}_2 = \frac{q(1-t_G)}{q(1-t_G)+(1-q)(1-t_B)} \geq q,$$

which contradicts (C.6). Since our observations in Steps 1 and 2 cannot jointly hold true, there cannot exist any equilibrium where the brown firm chooses the brown project. *Q.E.D.*

D The Capital Market with Green Lenders Only

In this section, we consider a capital market where all lenders are the green type. We prove that the same equilibrium outcome arises in both the static and dynamic versions of the lending game. In both the static and dynamic versions of the lending game, we will focus on perfect Bayesian equilibrium such that (a) all players play pure strategies, and (b) the tie-breaking rules (T1) and (T2) stated at the beginning of Appendix A are imposed.

In addition, we assume that there are at least two lenders in both periods of the dynamic lending game; hence, all lenders enjoy no market power. This assumption implies that all lenders earn zero financial profit in equilibrium, which simplifies the analysis. The assumption also levels the ground when comparing the dynamic and static lending games by guaranteeing that all lenders have no market power in both games.

We first make some preliminary observations. Assumptions 1 and 2 together imply:

$$\frac{1}{p_G} < \frac{1+\phi}{p_B}, \tag{D.1}$$

$$\frac{p_B - p_G}{p_G} < \phi, \tag{D.2}$$

$$\frac{1}{qp_G + (1-q)p_B} \leq \frac{1+q\phi}{p_B} \quad \forall q \in [0,1]. \tag{D.3}$$

See the end of Section A.2 for the proof of (D.1) and (D.2). (D.3) is trivial when $q = 0$. For $q \in (0, 1]$, the inequality in (A.3) is equivalent to

$$\frac{1}{qp_G + (1-q)p_B} \leq \frac{1+q\phi}{p_B} \quad \Longleftrightarrow \quad \frac{p_B - p_G}{p_G} \leq q\phi + (1-q)\frac{p_B}{p_G}\phi,$$

where $(p_B - p_G)/p_G < \phi$ by (D.2) and $\phi < (p_B/p_G)\phi$ because $p_B > p_G$. Thus, (D.3) follows from (D.2).

Proposition D.1. *Consider the static version of the lending game such that $q \in (0, 1)$, $n_G \geq 2$ and $n_B = 0$. Suppose that Assumption 1 and 2 hold.*

(i) There is an equilibrium such that both types of the firm choose the brown project if and only if

$$\frac{1}{p_B} \leq \overline{D}(\lambda_G) \tag{D.4}$$

(ii) There is an equilibrium such that both types of the firm choose the green project if and only if

$$\overline{D}(\lambda_B) < \frac{1}{p_G} \tag{D.5}$$

(iii) There is an equilibrium such that brown and green types of the firm respectively choose the brown and green projects if and only if

$$\overline{D}(\lambda_G) < \frac{1}{qp_G + (1-q)p_B} \leq \overline{D}(\lambda_B). \tag{D.6}$$

Proof. By Lemma B.1, all lenders earn zero financial profit in any equilibrium. Furthermore, all lenders' financial profit is zero only if one of the following three conditions hold:

$$D^* = \frac{1}{p_B} \leq \overline{D}(\lambda_G); \tag{D.7}$$

$$D^* = \frac{1}{p_G} > \overline{D}(\lambda_B); \tag{D.8}$$

$$D^* = \frac{1}{qp_G + (1-q)p_B} \in (\overline{D}(\lambda_G), \overline{D}(\lambda_B)]. \tag{D.9}$$

Both brown and green types choose the brown project in the case of (D.7), whereas both types choose the green project in the case of (D.8). Brown and green types respectively choose the brown and green projects in the case of (D.9). Thus, to prove the proposition, it suffices to show that (D.7) – (D.9) are also sufficient conditions for the existence of respective types of equilibria.

Sufficiency Part in (i): Suppose that (D.4) holds true, and consider the case where all green lenders offer $D^* = 1/p_B$ and then both types of the firm choose the brown project. It is straightforward that no player has an incentive to deviate. In particular, no lender can alter either firm type's project selection by undercutting D^*. This completes the proof of the sufficiency of (D.4).

Sufficiency Part in (ii): Suppose that (D.5) holds true, and consider the case where all green lenders offer $D^* = 1/p_G$ and both types of the firm choose the green project. In this case, all lenders obtain ϕ as their final payoff. To see that no green lender has an incentive to undercut D^*, suppose first that a green lender deviates and offers $D' < D^*$. Any deviation to $D' \in (\overline{D}(\lambda_B), D^*)$ is strictly unprofitable, because all such deviations only result in a negative financial profit without changing the firm's project selection. Thus, we will focus on the case $D' \leq \overline{D}(\lambda_B)$.

First, suppose $D' \leq \overline{D}(\lambda_G) < \overline{D}(\lambda_B) < D^*$. Then, both types of the firm would accept D' and then choose the brown project, yielding the payoff $p_B D' - 1$ to the deviating lender. However,

$$\begin{aligned}
p_B D' - 1 \leq p_B \overline{D}(\lambda_G) - 1 &< p_G(1+\phi)\overline{D}(\lambda_G) - 1 && \because \text{(D.1)} \\
&< p_G(1+\phi)\overline{D}(\lambda_B) - 1 && \because \overline{D}(\lambda_G) < \overline{D}(\lambda_B) \\
&< \phi && \because \text{(D.5)}
\end{aligned}$$

and thus, the deviation is not profitable.

Next, suppose that a lender deviates by offering $D' \in (\overline{D}(\lambda_G), \overline{D}(\lambda_B)] \subsetneq (\overline{D}(\lambda_G), 1/p_G)$. Then, the green and brown types of the firm respectively choose the green and brown projects after they accept D'. Thus, the deviating lender obtains $[qp_G + (1-q)p_B]D' - 1 + q\phi$ as final payoff. However,

$$
\begin{aligned}
[qp_G + (1-q)p_B]D' - 1 + q\phi &\leq [qp_G + (1-q)p_B]\overline{D}(\lambda_B) - 1 + q\phi && \because D' \leq \overline{D}(\lambda_B) \\
&< (1-q)\frac{p_B - p_G}{p_G} + q\phi && \because \text{(D.5)} \\
&< \phi && \because \text{(D.2)}
\end{aligned}
$$

and thus, the deviation is not profitable. In sum, no green lender has the incentive to deviate from D^*, which proves the sufficiency part in (ii).

Sufficiency Part in (iii): Suppose that (D.6) holds true, and consider the case where all lenders offer $D^* = 1/[qp_G + (1-q)p_B]$ and then the green and brown types of the firm respectively choose the green and brown projects after they accept D^*. Note that all lenders obtain $q\phi$ as their expected final payoff.

To show that no green lender has the incentive to undercut D^*, suppose that one of them deviates by offering $D' < D^*$. Any deviation to $D' \in (\overline{D}(\lambda_G), D^*)$ is strictly unprofitable because all such deviations only result in a negative financial profit without changing the firm's project selection. Thus, we focus on the case $D' \leq \overline{D}(\lambda_G)$, in which case both types of the firm accept D' and then choose the brown project. As a result, the deviating lender will obtain the payoff $p_B D' - 1$, where

$$
\begin{aligned}
p_B D' - 1 &\leq (1 + q\phi)(qp_G + (1-q)p_B)D' - 1 && \because \text{(D.3)} \\
&< 1 + q\phi - 1 = q\phi && \because D' < D^* = 1/(qp_G + (1-q)p_B).
\end{aligned}
$$

Thus, the payoff from deviation will be strictly lower than the lenders' equilibrium payoff. This completes the sufficiency part in (iii). *Q.E.D.*

The next proposition characterizes the equilibrium of the dynamic version of the lending game such that $n_G^1 \geq 2$, $n_G^2 \geq 2$, and $n_B^1 = n_B^2 = 0$. Note that the conditions (D.10) – (D.12) below coincide with (D.4) – (D.6) in Proposition D.1. Hence, when there are only green lenders in the market, granting a subset of green lenders a competitive advantage *à la* Stackelberg (in the sense that $n_1^G \geq 2$) cannot make any change to the equilibrium outcome of the game.

Proposition D.2. *Consider the dynamic version of the lending game such that* $q \in (0, 1)$*,* $n_G^1 \geq 2$*,* $n_G^2 \geq 2$*, and* $n_B^1 = n_B^2 = 0$*. Suppose that Assumption 1 and 2 hold.*

(i) There is an equilibrium such that both types of the firm choose the brown project if and only if

$$\frac{1}{p_B} \leq \overline{D}(\lambda_G). \tag{D.10}$$

(ii) There is an equilibrium such that both types of the firm choose the green project if and only if

$$\overline{D}(\lambda_B) < \frac{1}{p_G}. \tag{D.11}$$

(iii) There is an equilibrium such that brown and green types of the firm respectively choose the brown and green projects if and only if

$$\overline{D}(\lambda_G) < \frac{1}{qp_G + (1-q)p_B} \leq \overline{D}(\lambda_B). \tag{D.12}$$

Proof. The proof of (i) and (ii) is essentially identical to the proof of the corresponding parts of Proposition D.1. We focus on the part (iii) here. To prove the sufficiency of (D.12), simply consider the case where all lenders in both periods offer $D^* = 1/[qp_G + (1-q)p_B]$ and the green firm (respectively, the brown firm) accepts D^* in the second period and then chooses the green project (respectively, the brown project). It is straightforward that no player has any incentive to deviate, which proves that an equilibrium of interest always exists when (D.12) holds.

To prove the necessity of (D.12), suppose that there is an equilibrium such that green and brown types of the firm respectively invest in the brown and green projects although (D.12) fails. The firm accepts the lowest repayment offer on the equilibrium path. Hence, $\overline{D}(\lambda_G) < D^* = \min\{D_1^*, D_2^*\} \leq \overline{D}(\lambda_B)$ in any such an equilibrium.

There are two cases we will consider separately. First, consider the case $D^* = D_2^* < D_1^*$ or $D^* = D_1^* < D_2^*$ hold. In the former case, both types of the firm are financed in the second period; in the latter case, both of them are financed in the first period. In either case, all green lenders should financially break even, which implies

$$\overline{D}(\lambda_G) < D^* = \frac{1}{qp_G + (1-q)p_B} \leq \overline{D}(\lambda_B). \tag{D.13}$$

This shows the necessity of (D.12) for the case $D_1^* \neq D_2^*$.

In the remaining part of the proof, we will focus on the case $D^* = D_1^* = D_2^*$. The proof for the subcase such that $q_2^* = q$ is essentially identical to the previous paragraph. Hence, we will assume $q \neq q_2^*$ without loss. The firm is indifferent between D_1^* and D_2^*. But, the posterior belief q_2^* disagrees with the prior q in equilibrium only if $qt_G + (1-q)t_B > 0$ (i.e., only if the firm accepts D_1^* with positive probability). Furthermore, both D_1^* and D_2^* would induce green firm to invest in the green project and the brown firm to invest in the brown project, respectively, once accepted by the firm; thus, the total expected social value generated in equilibrium must be equal to $q\phi$ no matter when the firm borrows.

Suppose that the firm accepts D_1^* in equilibrium. Conditional on the event that the firm accepts D_1^*, the firm is green with probability $qt_G/[qt_G+(1-q)t_B]$ (and the firm is brown with the complementary probability). Thus, any lender from whom the firm accepts D_1^* ends up with a financial profit

$$\pi_1^* := \frac{qt_Gp_G + (1-q)t_Bp_B}{qt_G + (1-q)t_B} D^* - 1.$$

On the other hand, any lender from whom the firm accepts D_2^* earns financial profit of

$$\pi_2^* := [p_G q_2^* + p_B(1-q_2^*)]D^* - 1.$$

Again, by the standard argument of Bertrand competition,

$$\pi_1^* = \pi_2^* = 0 \quad \Longrightarrow \quad D^* = \frac{1}{p_G q_2^* + p_B(1-q_2^*)} = \frac{qt_G + (1-q)t_B}{qt_Gp_G + (1-q)t_Bp_B}. \tag{D.14}$$

Substituting $t_B = t_G$ in (D.14), we obtain $q = q_2^*$. The case such that $q = q_2^*$ has been already discussed before; hence, suppose $t_B \neq t_G$ without loss. In this case, the firm rejects D_1^* with positive probability (i.e., $t_G < 1$ or $t_B < 1$), and thus, q_2^* should satisfy Bayes' rule:

$$q_2^* = \frac{q(1-t_G)}{q(1-t_G) + (1-q)(1-t_B)}. \tag{D.15}$$

Substituting q_2^* in (D.14) with (D.15),

$$D^* = \frac{q(1-t_G) + (1-q)(1-t_B)}{p_G q(1-t_G) + p_B(1-q)(1-t_B)} = \frac{qt_G + (1-q)t_B}{qt_Gp_G + (1-q)t_Bp_B} \quad \Longleftrightarrow \quad t_B = t_G,$$

which contradicts the supposition $t_B \neq t_G$. This completes the proof of the necessity of (D.12). *Q.E.D.*