Uncertainty, Social Valuation, and Climate Change Policy

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How proactive should we be in the face of uncertainty?

To answer this question, we confront two uncertainty trade-offs:

- How much weight do we assign to:
  - best guesses
  - potentially bad outcomes

  when designing policy?
- Do we act now, or do we wait until we learn more?

Climate change challenge

"The economic consequences of many of the complex risks associated with climate change cannot, however, currently be quantified. ... these unquantified, poorly understood and often deeply uncertain risks can and should be included in economic evaluations and decision-making processes."

Rising, Tedesco, Piontek, Stainforth, Nature, 2022

Intertemporal marginal valuations

- are used in a variety of settings: public economics, environmental economics, and macroeconomics for policy assessment;
- help to navigate movements from suboptimal allocations to more efficient alternatives;
- provide interpretations of first-order conditions for optimal policies, including dynamic investment choice;
- support the construction of Pigouvian taxes aimed at implementing an optimal policy or a robust counterpart.

Their applications raise conceptual challenges that are often ignored in practice. I will outline a framework for exploring intertemporal marginal valuations and provide formulas that deconstruct the valuations into alternative interpretable components.

Ingredients

- I. Impose the controls (investments and other actions), and characterize the state variable interactions. Controls do not have to (robustly) optimal.
- II. Form stochastic (nonlinear) impulse responses.
- III. Construct a value function.
- IV. Deduce an asset pricing-type formula for the implied marginal valuation with marginal utility adjustments.
- V. Compute an alternative probability measure that adjusts for uncertainty aversion.
- VI. Introduce infrequent big changes as Poisson jumps with probabilities that depend on state variables.

Produce two additive deconstructions of the contributions to the marginal valuation.

I. Stochastic state evolution

Initially, consider diffusion dynamics:

$$dX_t = \mu(X_t)dt + \sigma(X_t)dW_t$$

where X is a Markov diffusion and W is a multivariate standard Brownian motion.

We will construct a stochastic impulse response of state vector that measures the impact of a marginal change in one of the initial states.

II. Stochastic responses

- Form the vector stochastic response process, Λ, that gives the marginal impact on future Xs of a marginal change in one of the initial states.
- Initialize the process at one of the coordinate vectors to specify the initial state of interest.
- Stochastic evolution that depends on X.

Characterizing Λ

- The process Λ^i evolves as:

$$d\Lambda_t^i = (\Lambda_t)' \frac{\partial \mu_i}{\partial x}(X_t)dt + (\Lambda_t)' \frac{\partial \sigma_i}{\partial x}(X_t)dW_t.$$

  where Λ^i is the i^{th} component of Λ.
- Stack all of the Λ^i's and study the joint dynamics (X, Λ).
- Linear VAR (vector autoregression) counterpart is obtained with a drift $\mu(x)$ that is linear in x and a Brownian exposure matrix σ that is constant.
- Λ is stochastic in general.

III. Value function

- Form a value function, V, associated with a discounted objective by:
  - imposing potentially suboptimal controls (for example, investments or given policy rules);
  - characterizing its dependence on state variables.
- Investigate marginal values, computed as partial derivatives, that capture small changes in endogenous state variables.

Climate model example: state variables

- Productive capital evolves as an AK model with investment adjustment costs.
- Knowledge stock evolves as second capital stock with R&D investment inputs.
- Temperature increases with emissions prior to the big technological discovery.

Technological success depends on the stock of knowledge and is a Poisson event that replaces fossil fuels with an entirely clean and economically viable alternative.

Climate model: production inputs

- capital
- fossil fuel-based energy

Temperature increases induce damages to the production of the consumption good.

Climate model example: production outputs

- consumption - is diminished by global warming
- investment in capital - increases future output
- investment in R&D - increases the stock of knowledge

Prudent choices for the investments and energy input depend on marginal valuations (partial derivatives of V). Two marginal valuations are relevant:

- social cost of climate change
- social value of research and development.

IV. An initial asset-pricing formula

Represent a partial derivative as an asset price:

$$\frac{\partial V}{\partial x}(X_0) \cdot \Lambda_0$$
$$= \delta \int_0^\infty \exp(-\delta t) E\left[\frac{\partial U}{\partial x}(X_t) \cdot \Lambda_t \mid X_0, \Lambda_0\right] dt.$$

where δ is the subjective rate of discount, and U is the utility contribution to the value function.

- Initializing Λ_0 with alternative coordinate vectors gives the derivatives of interest.
- $\frac{\partial U}{\partial x}(X_t)$ is marginal utility contribution in the future.
- Λ is the vector of stochastic impulse responses.

A first deconstruction of marginal values

Recall a partial derivative as an asset price:

$$
\begin{aligned}
&\frac{\partial V}{\partial x}(X_0) \cdot \Lambda_0 \\
&= \delta \int_0^\infty \exp(-\delta t) E \left[\frac{\partial U}{\partial x}(X_t) \cdot \Lambda_t \mid X_0, \Lambda_0 \right] dt.
\end{aligned}
$$

- the flow term:

$$
\frac{\partial U}{\partial x}(X_t) \cdot \Lambda_t
$$

  gives an additive deconstruction in terms of the alternative states where contribution j is:

$$
\delta \int_0^\infty \exp(-\delta t) \mathbb{E} \left[\frac{\partial U}{\partial x_j}(X_t) \Lambda_{j,t} dt \mid X_0, \Lambda_0 \right].
$$

Next explore two extensions

- Adjustments for broadly-based uncertainty concerns.
- Introduction of "big events" - damage realization, technology discovery.

Haunted by Hayek's forewarning

"Even if true scientists should recognize the limits of studying human behaviour, as long as the public has expectations, there will be people who pretend or believe that they can do more to meet popular demand than what is really in their power."

From Hayek's Nobel address (1974)

What are the uncertainties?

Four channels of uncertainty:

- productivity: capital investment today alters future output
- geosciences: CO_2 emissions today impact the future climate
- economics: climate change in the future alters economic opportunities and social well-being
- technology: research and development invested today may eventually lead to economically viable technologies

Two policy levers:

- reduce fossil fuel emissions
- invest in the discovery of new technologies that are clean replacements

Advances in decision theory under uncertainty

Decision theory seeks to develop and justify approaches that are "rational" — or better "prudent." It allows for a broad perspective on uncertainty.

Do not know:

- outcomes but probabilities are known - risk
- which among multiple probability models is best - ambiguity (prior uncertainty)
- ways in which a model might give flawed probabilistic predictions - misspecification (likelihood uncertainty)

Decision-maker preferences encode aversions to these uncertainties. The remainder of today's talk will focus on potential misspecification. There are counterpart methods for robust interpretations of "smooth ambiguity preferences."

What are the risks in our model?

- Brownian motions - locally normally distributed shocks
- Poisson events - discrete and one-off events

We relax full confidence in the probability specification and introduce robustness concerns.

Statisticians' wisdom

> *"Since all models are wrong, the scientist must be alert to what is importantly wrong. It is inappropriate to be concerned about mice when there are tigers abroad." Box (1976).*

> *"The very word "model" implies simplification and idealization. ... The construction of idealized representations that capture important stable aspects of such systems is, ..., a vital part of general scientific analysis and statistical models, especially substantive ones ... " Cox (1995).*

Axiomatic paper: "Making Decisions Under Model Misspecification" (*ReStud*, forthcoming) with Cerreia–Vioglio, Maccheroni, Marinacci

Decision problem

- Formulate a recursive max-min game where we:
  - minimize over the possible probability distortions subject to the penalization (with parameter ξ)
  - maximize over the possible decision processes.
- Sometimes there is an equivalent risk aversion interpretation, which we find hard to interpret for our application.
- As external analysts, we explore sensitivity of the minimizing distributions to the choice of ξ.

V. Uncertainty-adjusted probabilities

- Draw insights from derivative claims pricing and robust Bayesian theory to:
  - derive a "worst-case" probability distribution isolating where potential misspecification is most concerning;
  - use this as an uncertainty-adjusted probability measure for marginal valuation.
- Represent a partial derivative as an asset price:

$$
\begin{aligned}
&\frac{\partial V}{\partial x}(X_0) \cdot \Lambda_0 \\
&= \delta \int_0^\infty \exp(-\delta t) \widetilde{E} \left[\frac{\partial U}{\partial x}(X_t) \cdot \Lambda_t \mid X_0, \Lambda_0 \right] dt.
\end{aligned}
$$

where $\widetilde{E}$ is computed using the uncertainty-adjusted probability.

Uncertainty-adjusted probabilities: continued

- Both types of robustness lead to the construction of an uncertainty-adjusted probability measure.
- The deconstruction of the marginal valuations continues to apply using this probability measure.
- In spite of the mathematical equivalence, the altered probabilities are not intended to be the beliefs of the decision maker.

VI. Uncertain big abrupt changes in our climate-economics example

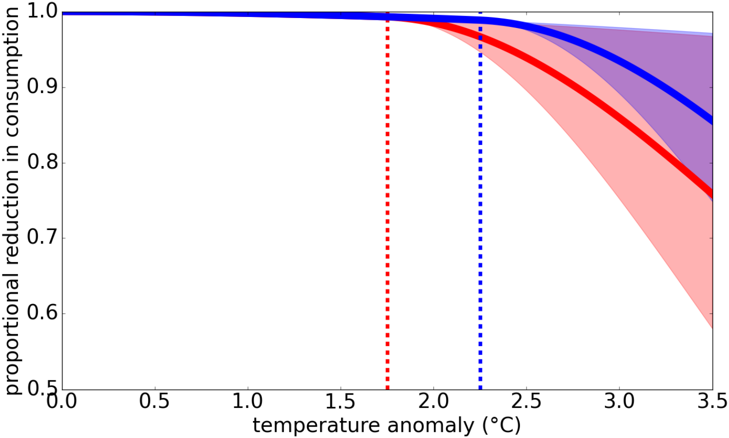
- Tipping point in the climate/economic system reveals damages for further global warming: there are many possible curves and one gets revealed
- Technological breakthrough that eliminates the need for dirty energy input

Endogenous intensities: the likelihood of the first change is enhanced by temperature increases, and the likelihood of second by knowledge stock. Note: an intensity times a short elapsed time interval gives the the corresponding approximate jump probability.

Stochastic model of damages

- Jump intensity increases substantially over the temperature anomaly degree interval $[1.5, 2.5]$.
- At the time of the jump, the damage curvature from that point forward is revealed where the tail curvature coefficient takes on one of twenty values.

Damage curves



Range of possible damage curves for two cases with different jump thresholds.

Including jumps in our analysis

Analyze the problem from a "pre-jump" perspective:

- Let $\mathcal{J}^\ell(x)$ denote the jump intensity to jump type ℓ, for $\ell = 1, 2, ..., L$.
- Let V^ℓ denote the state dependent continuation value for type ℓ.

Jump contributions

- Alter discount rate to adjust for a damage curve revelation jump or a technology discovery jump:

$$\delta + \sum_{\ell=1}^{L} \mathcal{J}^{\ell}(X_t).$$

- Additional flow contributions:
  - marginal impact of a jump:

$$\Lambda_t \cdot \sum_{\ell=1}^{L} \left[\frac{\partial \mathcal{J}^{\ell}}{\partial x}(X_t) \right] \left[V^{\ell}(X_t) - V(X_t) \right];$$

  - marginal impact when you jump:

$$\Lambda_t \cdot \sum_{\ell=1}^{L} \mathcal{J}^{\ell}(X_t) \left[\frac{\partial V^{\ell}}{\partial x}(X_t) \right].$$

Adjust for uncertainty aversion using the implied uncertainty-adjusted probability distribution.

A second decomposition

Compute marginal valuations implied by each of three contributions to the marginal flows:

- marginal utility contribution

$$\frac{\partial U}{\partial x}(X_t) \cdot \Lambda_t;$$

- marginal impact of jumps that depend on endogenous states:

$$\Lambda_t \cdot \sum_{\ell=1}^{L} \left[\frac{\partial \mathcal{J}^\ell}{\partial x}(X_t) \right] \left[V^\ell(X_t) - V(X_t) \right];$$

- marginal impact when you jump:

$$\Lambda_t \cdot \sum_{\ell=1}^{L} \mathcal{J}^\ell(X_t) \left[\frac{\partial V^\ell}{\partial x}(X_t) \right].$$

Robustness adjustment with jumps

- Introduce multiplicative adjustments to the intensities set subject to the relative entropy penalties:
  - for each ℓ, replace $\mathcal{J}^{\ell}$ with $\mathcal{J}^{\ell} g^{\ell}$, where the g^{ℓ}'s are deduced by minimizing
- Include uncertainty-adjustments in the continuation value functions.
- Construct the resulting exogenous probability to use for marginal valuations and decompositions.

Findings

Our initial research shows that:

- the uncertain timing of R&D success is the most significant contributor to uncertainty for climate-economics policy;
- this source of uncertainty leads to doing more green R&D investment;
- reduce emissions in the short term to allow for R&D to have a chance to be successful, even though this response is less sensitive to the degree of uncertainty aversion. In general, the response increases after the damage information event is realized.

Uncertainty-adjusted probability assessments

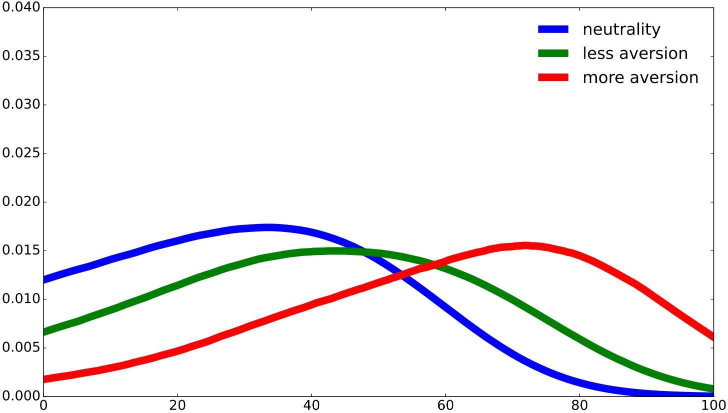


Figure: Jump time densities for a technology discovery

Why more proactive investment in R&D?

Two counteracting forces:

- adopt a more cautious view of when the technological success will be realized
- achieve a greater benefit to a successful discovery because of the substantial reduction in the exposure to uncertainty.

The second force is captured by changes in the marginal impact of the technology as measured by

$$\left[\frac{\partial \mathcal{J}^{\ell}}{\partial x}(X_t)\right]\left[V^{\ell}(X_t) - V(X_t)\right].$$

We find that the second force dominates in our computations over a potentially relevant range of uncertainty aversion.

Only for some magnitudes of uncertainty aversion does R&D investment becomes more proactive

	knowledge stock valuation	R&D investment / output
$\xi = \infty$	44.6	.008
$\xi = .10$	63.4	.015
$\xi = .05$	87.9	.028
$\xi = .01$	88.3	.030
$\xi = .009$	82.6	.026
$\xi = .008$	75.0	.021
$\xi = .007$	66.4	.017
$\xi = .006$	56.2	.012
$\xi = .005$	44.4	.007

Social value of R&D and R&D investment as a function of the robustness parameter, ξ.

Conclusions

- Sometimes the best response to uncertainty is to be more proactive.
- More transparent communication of scientific uncertainty can lead to better policy.
- The extensions and refinements of uncertainty quantification that I described have more general applicability to the study of dynamic models.
- This research is part of a larger agenda to explore the impacts of uncertainty on both private and public sector decision-making.