Decarbonizing Portfolios: A Framework for Integrating Emission Constraints into Optimal Investment

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Abstract

Reducing the carbon footprint of financial portfolios has become a central challenge in climate finance. A variety of metrics are available to measure portfolio decarbonization, ranging from straightforward year-on-year emission reduction targets to more sophisticated, forward-looking indicators such as Net-Zero frameworks or the Implied Temperature Rise (ITR) score. In this paper, we examine the implications of incorporating such decarbonization constraints into optimal portfolio choice. We propose a methodology to formally integrate portfolio-owned carbon emissions into the dynamic optimization problem and highlight the conceptual distinction between emission reduction targets and cumulative emission constraints. By formalizing these within a unified framework, we highlight their conceptual differences and analyse how each influences optimal investment strategies and long-term sustainability outcomes.

1 Introduction

The integration of climate considerations into financial decision-making has become increasingly important as investors, regulators, and policymakers seek to decarbonize their portfolios and align capital allocation with a net-zero pathway. Investors are under growing pressure to reduce the climate footprint of their portfolios, both to manage transition risks and to contribute to the achievement of international climate targets such as those outlined in the Paris Agreement. In response, a range of portfolio decarbonization metrics has been developed, from simple measures based on reported carbon emissions to forward-looking indicators that attempt to capture the trajectory of firms and portfolios with respect to climate goals. While these tools provide practical benchmarks for assessing climate performance, their integration into formal models of portfolio optimization remains limited, raising fundamental questions about how different types of emission constraints alter investment behaviour and long-term outcomes.

As outlined by Leote de Carvalho et al. (2024), two principal investment frameworks currently dominate the field. The first are the Paris-Aligned Benchmark (PAB) type approach, which builds on the regulatory standards introduced by the European Commission. This framework focuses on reducing the carbon intensity of benchmark indexes, by setting effective carbon-intensity constraint, that include:

- A minimum 50% reduction in greenhouse gas (GHG) emissions intensity relative to the parent benchmark.
- An annual 7% reduction in carbon intensity.
- Exclusion of companies with ties to fossil fuel activities.

The intensity-reduction pathway is generally assumed to be consistent with the emission cuts required to limit global warming to 1.5°C, as set out in the Paris Agreement. Yet this approach has been criticized for its narrow focus on emissions, which may result in the exclusion of firms that are critical to the low-carbon transition. As shown by Roncalli (2024), applying such criteria can produce portfolios with low carbon intensity but a sectoral composition that fails to capture the real-economy requirements of the transition.

The second initiative is commonly referred to as Net-Zero investing. This concept emphasizes broad alignment with climate goals and includes companies that are actively decarbonizing, as well as those whose products are essential for the transition to a low-carbon economy. It adopts a scenario-based view of decarbonization, where interdependencies between companies must be considered. Net-Zero investing also places strong emphasis on shareholder engagement, which has been shown to reduce firms' downside risks Hoepner et al. (2024). While this general framework is conceptually rich, it is too complex for our purposes. Therefore, we simplify it to a narrower definition of alignment. In our context, alignment refers to matching the emissions of a portfolio or asset with those projected in a specific climate scenario Institut Louis Bachelier et al. (2024). A common quantitative metric for this is the difference between the portfolio's projected future emissions and those of a scenario-based benchmark PAT (2021). This difference can be translated into a temperature score (often refereed to as the Implied Temperature Rise (ITR) score); however, recent research suggests that a simpler and more robust approach is to focus on the difference in cumulative emissions over time Bouchet (2024).

These two initiatives correspond to two distinct types of constraints. The first is a constraint on the emission trajectory, which monitors year-on-year variations in portfolio emissions. It requires the portfolio to follow a predefined path of annual reductions (aligned with benchmark trajectories or exogenously fixed as for the PAB framework). The second is a constraint on cumulative emissions, which imposes a carbon budget over a given time horizon. This ensures that the total emissions of the portfolio remain within the limits defined by a climate scenario or target. The academic literature on how to incorporate the first type of constraint—emission trajectory constraints—is relatively well developed Bolton et al. (2022); Lazanas et al. (2025); Slimane et al. (2023). This type of constraint typically does not require forecasting future emissions and can be integrated into most index-tracking portfolio strategies.

Historically, the implementation of cumulative emissions constraints within a portfolio framework has been limited by the lack of reliable data. Specifically, applying such constraints requires credible forecasts of firm-level emissions and scenario-linked benchmarks—information that was previously scarce or inconsistent. However, recent developments have improved data availability. The growing number of publicly disclosed company-level emission targets, along with advances in climate scenario modelling—such as those provided by the Network for Greening the Financial System (NGFS) NGFS (2024)—now make it increasingly feasible to construct robust forecasts of firm-level emissions Bhattacharya et al. (2025) and corresponding benchmark trajectories. This progress opens the door to more sophisticated approaches that account for cumulative emissions over time. This shift motivates the need for a quantitative framework capable of incorporating cumulative constraints dynamically and compare them to emission trajectory constraints.

Building on this motivation, our paper develops a framework to determine the optimal portfolio under explicit carbon emission constraints. An optimal portfolio is defined as the portfolio that maximizes expected terminal utility. We distinguish between two approaches to portfolio decarbonization: an emission trajectory constraint and cumulative emission constraints. While both approaches aim decarbonize portfolios, we have seen that they differ in their economic interpretation and in the incentives they create for investors. By formalizing these constraints within a dynamic optimization setting, we highlight their conceptual differences and quantify the resulting impact on optimal portfolio allocations. Our analysis provides insights into how the choice of constraint influences portfolio allocation choices and the pace of decarbonization. This work contributes to the literature on dynamic portfolio allocation under sustainability constraints, and to the growing body of research on portfolio alignment with climate scenarios.

The remainder of this paper is organized as follows: the next section reviews the literature on the topic of sustainability targets and portfolio allocation. Section 3 formalizes the notations and concepts used throughout the paper. Section 4 provides details on the methodology used to define the emission trajectory and cumulative emission constraints. Section 5 provides a numerical application and Section 6 concludes.

2 Related literature

The integration of carbon-related constraints into portfolio construction is a well-established topic in the practitioner literature. A seminal contribution is Andersson et al. (2016), which demonstrates the impact of reducing a portfolio's carbon footprint on tracking error. In Lazanas et al. (2025), the authors explore the challenges of building Net-Zero aligned portfolios, introducing forward-looking emissions and time-evolving climate budgets. They emphasize the importance of credible, region- and sector-specific emissions trajectories, particularly for comparing a portfolio's future emissions with those of its benchmark. An alternative approach to incorporating sustainability considerations is factor tilting, as described in Riposo and Wang (2023). This method involves overweighting or under-weighting assets based on their exposure to specific risk factors. It is particularly suited to index construction, given its scalability and control over data inputs. However, factor tilting does not allow for the integration of explicit constraints—such as guaranteed emission reductions—and therefore cannot ensure alignment with specific climate targets.

A related approach is presented in Bolton et al. (2022), where the authors propose two strategies for portfolio decarbonization under a carbon trajectory constraint, while minimizing tracking error. The first strategy is exclusion-based, progressively removing the highest-emitting companies within each sector. The second strategy relies on re-weighting, adjusting asset allocations to favour lower-emission firms while maintaining sectoral exposure. Both methods aim to reduce portfolio emissions without significantly deviating from the benchmark, offering practical tools for climate-aware portfolio construction.

Another example of portfolio optimization under explicit carbon constraints is provided in Le Guenedal and Roncalli (2022), where the authors examine a range of climate-related metrics and assess their impact when used as constraints in a mean-variance optimization framework. Building on this approach, Slimane et al. (2023) introduces an innovative methodology known as the core-satellite framework. This strategy acknowledges that a credible Net-Zero investment approach must address two interconnected dimensions: decarbonization and transition financing. These objectives can diverge, requiring a dual-focus strategy that separately optimizes for emissions reduction and capital allocation toward sectors enabling the low-carbon transition.

A more theoretical perspective is offered in Azzone et al. (2024), where the authors analyse the impact of integrating Environmental, Social and Governance (ESG) scores—treated as an ESG mandate—on the Tracking Error Variance (TEV) frontier. This work is closely related to Spiegeleer et al. (2023) and Di Zio et al. (2023), who show that, under equilibrium conditions, an ESG mandate can result in a negative ESG premium. In other words, investors may accept lower expected returns for the same level of risk when investing in companies with higher ESG scores. While these studies provide valuable insights, they remain static in nature: portfolio optimization is performed at a single point in time, without accounting for uncertainty or the dynamic evolution of data.

Our work is closely related to the studies by Korn and Nurkanovic (2023) and Chen et al. (2025), which integrate sustainability-linked constraints into dynamic portfolio optimization frameworks. In Korn and Nurkanovic (2023), the focus is on ESG-related constraints, particularly in the context of life insurers and large institutional portfolios. The authors highlight how regulatory risks and liquidity limitations can restrict the ability to adjust portfolio weights. Follow-up studies extend this framework to alternative utility functions Korn and Nurkanović (2025) and incorporate scenario-based analysis Korn (2025). In Chen et al. (2025), the authors examine portfolio optimization under carbon risk constraints within an expected utility framework. They derive closed-form solutions for asset weights under specific utility assumptions, with a focus on the relative allocation between green and brown assets rather than the overall portfolio structure.

Our study builds on this literature by incorporating explicit portfolio-owned emissions—rather than ESG scores—and simulating forward-looking emissions data. This allows us to explore the implications of different decarbonization constraints in a dynamic setting.

3 Formalization and notation

In this section, we introduce and formalize the notation useful to define the portfolio-owned emissions, and the benchmark emissions.

3.1 Portfolio level carbon emissions

We denote $E_t^i(j)$ the carbon emissions of firm i at time t for scope $j \in \{\text{Scope 1}, \text{Scope 2}, \text{Scope 3}\}$, measured in tCO2-eq/yr. We define the cumulative emissions $CE_{t_0,t}^i(j), t \geq t_0$ as the total of carbon emissions emitted by firm i for scope j between t_0 and t. It is defined as follow:

$$CE_i^j(t_0, t) = \sum_{\tau=t_0}^{t} E_i^j(\tau)$$

Furthermore, we define the carbon intensity $CI_t^i(j)$ as the normalization of the carbon emission by an output measure. This output can be physical (ton of steel, MWh) or monetary (revenues, market capitalization, enterprise value including cash (EVIC)). We define the output as $Y_i(t)$

$$CI_t^i(j) = \frac{E_t^i(j)}{Y_t^i}$$

Carbon intensity is additive for the scopes and $CI_t^i = CI_t^i(\text{Scope 1}) + CI_t^i(\text{Scope 2}) + CI_t^i(\text{Scope 3})$.

Now, let consider a portfolio P composed of d assets, that has a wealth of X_t at time t. We define $\pi_t = (\pi_t^1, ..., \pi_t^d)$ as the portfolio process, describing the share invested in each assets. Following Le Guenedal and Roncalli (2022), the carbon emissions associated to this portfolio are:

$$E_t^\pi(j) = \sum_{i=1}^{d} \pi_t^i X_t \frac{E_t^i(j)}{\text{EVIC}_t^i}$$

with $\pi_t^i X_t$ the dollar value invested in asset i at time t and EVIC_t^i the enterprise value including cash (EVIC) of asset i. We define the carbon intensity to EVIC of asset i at time t for Scope j

as:

$$I_t^i(j) = \frac{E_t^i(j)}{\mathrm{EVIC}_t^i}$$

In the next section, we define the notation needed to describe the emission benchmark, necessary to compute the emission target.

3.2 Benchmark emissions and alignment

A portfolio carbon benchmark, or reference trajectory, defines the emission pathway that an asset or portfolio is expected to follow in order to remain consistent with a given climate scenario. It is usually expressed in absolute emission or intensity and is specific to a certain scenario. The benchmark emissions at time t for scope j, asset i and scenario s is denoted by $\bar{E}_t^i(j,s)$. We also define cumulative reference emissions:

$$\bar{CE}_{t_0,T}^i(j,s) = \sum_{t=t_0}^{T} \bar{E}_t^i(j,s)$$

Asset-(or portfolio-)specific reference trajectories can be set exogenously (as in the PAB framework) or derived from overall emissions scenarios, provided for example by Integrated Asset Models (IAM). We define a scenario by a trajectory of future carbon emissions, expressed in tCO2-eq/yr. We assume that these trajectories are scope, region and sector specific. We call the scenario emissions at time t for region r, sector l, scope j and scenario s: $SE_t^{r,l}(j,s)$. There are several approaches to derive an asset-specific references trajectories : the fair-share approach, the convergence approach or the rate of reduction approach.

Fair-share approach This approach is, theoratically, the most satisfactory and works well with absolute emissions. The goal is to allocate emissions from an "overall" budget to an asset (or portfolio), based on some value representing the share (market-share, size, etc.) of that asset. For example, we assume that asset i is located in region r and sector l. Then, the emission benchmark can be expressed as:

$$\bar{E}_t^i(j,k) = \frac{Y_t^i}{\sum_{k\in r, k\in s} Y_t^i} SE_t^{r,l}(j,s)$$

In practise, determining the fair-share ratio raises various questions: what variable to use to define a market share ?, How to determine the region of an asset ?, etc. This makes the application of this method complex and subject to numerous methodological assumptions.

Convergence approach Here, we assume that the carbon intensity of all assets convergence to a specific value at a certain time. We assume that asset i is located in region r and sector s. Moreover, we assume a current date t_0 as well as a final date T, time at which we want to achieve $SEI_t^{r,l}(j,s) = \frac{SE_t^{r,l}(j,s)}{Y_t^{r,l}(s)}$ (the carbon intensity from the scenarios). The benchmark carbon intensity at time t is given by:

$$\bar{CI}_t^i(j,s) = CI_{t_0}^i(j) + (t-t_0)\frac{SEI_t^{r,l}(j,s)(T) - CI_{t_0}^i(j)}{T-t_0}$$

To obtain the emission benchmark, you need additional information on the normalizing factor to transform carbon intensity into emissions.

Rate-of-reduction With this approach, we use the rate of reduction observed in the scenario and apply the same rate to the current value. This methodology can be applied to either absolute value or intensity. We write the formula for absolute emission. We define a rate of reduction between t_0 and t :

$$R^{r,l}_{t_0,t}(j,s) = \frac{SE^{r,l}_t(j,s)}{SE^{r,l}_{t_0}(j,s)} - 1$$

The asset-specific benchmark can be defined as:

$$\bar{E}^i_t(j) = E^i_{t_0}(j) * (1 - R^{r,l}_{t_0,t}(j,s))$$

For the case of a PAB, the rate of reduction approach is used, with an initial rate of reduction of 50%, followed by an annual 7% reduction.

We see that the last two methods use the "latest available" data (ie: emission or carbon intensity at time t_0), making them subject to data inconsistencies. Moreover, the rate of reduction approach will disadvantage assets with reduced emissions (compared to their peers) as they will have the same rate of reductions. On the contrary, the convergence approach, given that it is based on carbon intensity, cannot ensure that a specific carbon budget is attained.

The construction of benchmark emission trajectories is a complex process that depends on numerous methodological assumptions, some of which are discussed in Bouchet (2024). While the focus of this paper is not to detail how these benchmarks are constructed, it is nonetheless important to understand how they are produced and how they connect to underlying climate scenarios. In our analysis, we abstract away from distinctions such as emission scopes and scenario indices, treating them as fixed for simplicity.

4 Dynamic portfolio allocation under emission constraint

4.1 Overall setting

We first describe the overall general framework, before considering the two specific constraints: on the emission trajectory and on the cumulative emissions. We suppose a investor who wants to maximize utility from wealth at terminal time T. There are d risky assets on the market and one risk-free asset. Furthermore, we consider a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$, equipped with a right-continuous filtration $\mathbb{F} = (\mathcal{F}_t)_{t\in[0,T]}$ an d-dimensional Brownian motion $(W_t)_{t\in[0,T]}$ and an d-dimensional Brownian motion $(B_t)_{t\in[0,T]}$ both adapted to the filtration $\mathbb{F}$. We assume a risk free assets as well as d risky assets, whose price dynamics are given by:

$$dS^0_t = rS^0_t dt,\ S^0_0 = 1$$

$$dS^i_t = \mu_i S^i_t dt + S^i_t \sum_{j=1}^{d} \sigma_{i,j} dW^j_t, S^i_0 = s_i, i = 1, ..., d$$

with $(W_t)_{t\in[0,T]}$ a standard d-dimensional Brownian motion, with $s_i > 0, i \in [1, \dots, d]$ and $\Sigma^W = (\sigma_{i,j})_{i,j\in[1,\dots,d]}$ where $\Sigma^W(\Sigma^W)^\top$ is positive definite. We define $\pi_t = (\pi^1_t, ..., \pi^d_t)$ as the portfolio process, describing the share invested in risky assets. Furthermore, we assume that the intensity is stochastic, with [1]:

$$dI^i_t = b_i I^i_t dt + I^i_t \sum_{j=1}^{d} \eta_{i,j} dB^i_t$$

[1]Note that for simplicity, we assume that the intensity is the sum of the intensity of all scopes and we ignore the index for the scope. The emission intensity of the risk-free asset is equal to 0.

We assume that all Brownian motions can be correlated through the covariance matrix:

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}^W & \boldsymbol{\Sigma}^{WB} \\ (\boldsymbol{\Sigma}^{WB})^\top & \boldsymbol{\Sigma}^B \end{bmatrix}$$

To summarize, we have the following parameters:

- $\boldsymbol{\pi}_t \in \mathbb{R}^d$: portfolio weights,
- r expected return of the riskless asset
- $\boldsymbol{\mu} \in \mathbb{R}^d$, $\boldsymbol{\Sigma}^{\boldsymbol{W}} \in \mathbb{R}^{d\times d}$: expected returns and volatility matrix of risky assets,
- $\boldsymbol{b} \in \mathbb{R}^d$, $\boldsymbol{\Sigma}^B \in \mathbb{R}^{d\times d}$: drift and volatility of emission intensity,
- $\boldsymbol{W}_t \in \mathbb{R}^d$: Brownian motions for risky assets,
- $\boldsymbol{W}_t^I \in \mathbb{R}^d$: Brownian motions for emission intensity,
- $\boldsymbol{\Sigma}^{\boldsymbol{WB}} \in \mathbb{R}^{d\times d}$: correlation matrix between $\boldsymbol{W}_t$ and $\boldsymbol{B_t}$, composed of elements $\rho_{i,j}$.

Next, we can define the wealth process of the portfolio:

$$dX_t^\pi = X_t^\pi \left(rdt + \boldsymbol{\pi}_t^\top (\boldsymbol{\mu} - r\mathbf{1})dt + \boldsymbol{\pi}_t^\top \boldsymbol{\Sigma}^{\boldsymbol{W}} d\boldsymbol{W}_t \right)$$

Similarly, we can derive the financed emissions of the portfolio, dependant on the wealth of the portfolio, the portfolio process and the emission intensity process:

$$E_t^\pi = X_t \boldsymbol{\pi}_t^\top \boldsymbol{I_t}$$

Finally, the portfolio cumulative emissions up to time t are defined as:

$$CE_{t_0,t}^\pi = \sum_{\tau=t_0}^{t} E_\tau^\pi$$

Using, this, we define the two following constraints:

- **Constraint on the emission trajectory** : the probability the the next-period emissions are above the target emission is lower than a specific threshold

$$\mathbb{P}(E_{t+\tau}^\pi > \bar{E}_{t+\tau} | \mathcal{F}_t) \leq \alpha, \forall t \in [0, T - \tau] \tag{1}$$

- **Constraint on the cumulative emissions** : the probability the total cumulative portfolio emissions are above the target emission budget is lower than a specific threshold

$$\mathbb{P}(CE_{0,T}^\pi > \bar{CE}_{0,T}^\pi | \mathcal{F}_t) \leq \alpha, \forall t \in [0, T] \tag{2}$$

where $\bar{E}_{t+\tau}$ (resp. $\bar{CE}_{t_0,T}$) are the target emissions (resp. cumulative target emissions).

4.2 Deriving the portfolio owned emission dynamic

In order to characterize these constraints, we start by deriving the process of the portfolio owned emissions. We define $\mathcal{E}_t = \boldsymbol{\pi}_t^\top \boldsymbol{I}_t$ and:

$$E_t^\pi = X_t \mathcal{E}_t$$

The dynamics of $\mathcal{E}_t$ are:

$$d\mathcal{E}_t = \boldsymbol{\pi}_t^\top d\boldsymbol{I}_t = \boldsymbol{\pi}_t^\top \mathrm{diag}(\boldsymbol{I}_t)(\boldsymbol{b}dt + \boldsymbol{\Sigma}^B d\boldsymbol{W}_t^I)$$

The dynamics of E_t^π are:

$$\begin{aligned} dE_t^\pi &= X_t d\mathcal{E}_t + \mathcal{E}_t dX_t + dX_t d\mathcal{E}_t \\ &= X_t \boldsymbol{\pi}_t^\top \mathrm{diag}(\boldsymbol{I}_t)(\boldsymbol{b}dt + \boldsymbol{\Sigma}^B d\boldsymbol{B}_t) + \mathcal{E}_t X_t \left[rdt + \boldsymbol{\pi}_t^\top (\boldsymbol{\mu} - r\boldsymbol{1})dt + \boldsymbol{\pi}_t^\top \boldsymbol{\Sigma}^W d\boldsymbol{W}_t \right] \\ &\quad + X_t \boldsymbol{\pi}_t^\top \mathrm{diag}(\boldsymbol{I}_t) \boldsymbol{\Sigma}^{\boldsymbol{WB}} (\boldsymbol{\Sigma}^W)^\top \boldsymbol{\Sigma}^B \boldsymbol{\pi}_t dt \end{aligned}$$

Define the drift and volatility as follows:

$$\begin{aligned} \mu_{\boldsymbol{\pi}_t}^{\mathcal{E}} &= r + \boldsymbol{\pi}_t^\top (\boldsymbol{\mu} - r\boldsymbol{1}) + \frac{\boldsymbol{\pi}_t^\top \mathrm{diag}(\boldsymbol{I}_t)}{\boldsymbol{\pi}_t^\top \boldsymbol{I}_t} \boldsymbol{b} + \frac{\boldsymbol{\pi}_t^\top \mathrm{diag}(\boldsymbol{I}_t)}{\boldsymbol{\pi}_t^\top \boldsymbol{I}_t} \boldsymbol{\Sigma}^{WB} (\boldsymbol{\Sigma}^{\boldsymbol{W}})^\top \boldsymbol{\Sigma}^B \boldsymbol{\pi}_t \\ (\sigma_{\boldsymbol{\pi}_t}^{\mathcal{E}})^2 &= \boldsymbol{\pi}_t^\top \boldsymbol{\Sigma}^W (\boldsymbol{\Sigma}^{\boldsymbol{W}})^\top \boldsymbol{\pi}_t + \left(\frac{\boldsymbol{\pi}_t^\top \mathrm{diag}(\boldsymbol{I}_t)}{\boldsymbol{\pi}_t^\top \boldsymbol{I}_t} \right) \boldsymbol{\Sigma}^B (\boldsymbol{\Sigma}^{\boldsymbol{B}})^\top \left(\frac{\mathrm{diag}(\boldsymbol{I}_t)\boldsymbol{\pi}_t}{\boldsymbol{\pi}_t^\top \boldsymbol{I}_t} \right) + 2\boldsymbol{\pi}_t^\top \boldsymbol{\Sigma}^W \boldsymbol{\Sigma}^{WB} (\boldsymbol{\Sigma}^B)^\top \left(\frac{\mathrm{diag}(\boldsymbol{I}_t)\boldsymbol{\pi}_t}{\boldsymbol{\pi}_t^\top \boldsymbol{I}_t} \right) \\ \sigma_{\pi_t}^{\mathcal{E}} d\tilde{W}_t &= \boldsymbol{\pi}_{\boldsymbol{t}}^\top \boldsymbol{\Sigma}^W d\boldsymbol{W}_t + \frac{\boldsymbol{\pi}_t^\top \mathrm{diag}(\boldsymbol{I}_t)}{\boldsymbol{\pi}_t^\top \boldsymbol{I}_t} \boldsymbol{\Sigma}^B d\boldsymbol{B}_t \end{aligned}$$

The SDE becomes:

$$dE_t^\pi = E_t^\pi \left[\mu_{\boldsymbol{\pi}_t}^{\mathcal{E}} dt + \sigma_{\boldsymbol{\pi}_t}^{\mathcal{E}} d\tilde{W}_t \right]$$

So, with $E_0^\pi > 0$:

$$E_t^\pi = X_0 I_0 \exp \left(\int_0^t \pi_s (\mu_{\pi_s}^{\mathcal{E}} - \frac{1}{2} (\sigma_{\pi_s}^{\mathcal{E}})^2) ds + \int_0^t \pi_s \sigma_{\pi_s}^{\mathcal{E}} d\tilde{W}_s \right)$$

This implies, for any $\tau > 0$:

$$dE_{t+\tau}^\pi = X_t I_t \exp \left(\int_t^{t+\tau} \pi_s \left(\mu_{\pi_s}^{\mathcal{E}} - \frac{1}{2} (\sigma_{\pi_s}^{\mathcal{E}})^2 \right) ds + \int_t^{t+\tau} \pi_s \sigma_{\pi_s}^{\mathcal{E}} d\tilde{W}_s \right)$$

For given $\tau > 0$, $\pi_t \in \mathbb{R}$ and assuming that π_t is held constant over the interval $[t, t+\tau]$ as in Yiu (2004) and Cuoco et al. (2008), the solution to this SDE is :

$$E_{t+\tau}(E_t^\pi, \pi_t) = E_t^\pi \exp \left[\left(\mu_{\pi_t}^{\mathcal{E}} - \frac{1}{2} (\sigma_{\pi_t}^{\mathcal{E}})^2 \right) \tau + \sigma_{\pi_t}^{\mathcal{E}} [\tilde{W}_{t+\tau} - \tilde{W}_t] \right]$$

So $E_{t+\tau}(E_t^\pi, \pi_t)$ are the financed emissions of the portfolio at time $t+\tau$ if the portfolio weights are kept constant between t and $t+\tau$. To avoid confusion with the "real" portfolio owned emissions at time $t+\tau$, we express them as a function of the portfolio process and emissions at time t. This expression allows us to compute projection of futur portfolio owned emissions, as a function of the portfolio holdings and the portfolio owned emissions at time t.

4.3 Reformulation of the constraints using the expression of the portfolio-owned emission

4.3.1 Constraint on the emission trajectory

We start by looking at the constraint 1, where we replace the value of the portfolio owned emissions at time $t+\tau$ by their estimation given information at time t. This steps allows us to construct an explicit formula for the constraint. Indeed, we can express this probability as:

$$
\begin{aligned}
&\mathbb{P}(E_{t+\tau}(E_t^\pi,\pi_t) > \bar{E}_{t+\tau}|\mathcal{F}_t) = \\
&\mathbb{P}(\exp\left[\left(\mu_{\pi_t}^{\mathcal{E}} - \frac{1}{2}(\sigma_{\pi_t}^{\mathcal{E}})^2\right)\tau + \sigma_{\pi_t}^{\mathcal{E}}[\tilde{W}_{t+\tau} - \tilde{W}_t]\right]) \geq \frac{\bar{E}_{t+\tau}}{E_t^\pi}|\mathcal{F}_t) = \\
&\mathbb{P}(\sigma_{\pi_t}^{\mathcal{E}}[\bar{W}_{t+\tau} - \bar{W}_t]) \geq \log\left(\frac{\bar{E}_{t+\tau}}{E_t^\pi}\right) - \left(\mu_{\pi_t}^{\mathcal{E}} - \frac{1}{2}(\sigma_{\pi_t}^{\mathcal{E}})^2\right)\tau|\mathcal{F}_t) = \\
&1 - \Phi\left(\frac{\log\left(\frac{\bar{E}_{t+\tau}}{E_t^\pi}\right) - \left(\mu_{\pi_t}^{\mathcal{E}} - \frac{1}{2}(\sigma_{\pi_t}^{\mathcal{E}})^2\right)\tau}{\sigma_{\pi_t}^{\mathcal{E}}\sqrt{\tau}}\right)
\end{aligned}
$$

The chance constraint then becomes:

$$
\begin{aligned}
&\mathbb{P}(E_{t+\tau}(E_t^\pi,\pi_t) > \bar{E}_{t+\tau}|\mathcal{F}_t) \leq \alpha \Leftrightarrow \\
&1 - \Phi\left(\frac{\log\left(\frac{\bar{E}_{t+\tau}}{E_t^\pi}\right) - \left(\mu_{\pi_t}^{\mathcal{E}} - \frac{1}{2}(\sigma_{\pi_t}^{\mathcal{E}})^2\right)\tau}{\sigma_{\pi_t}^{\mathcal{E}}\sqrt{\tau}}\right) \leq \alpha \Leftrightarrow \\
&E_t^\pi \exp\left[\left(\mu_{\pi_t}^{\mathcal{E}} - \frac{1}{2}(\sigma_{\pi_t}^{\mathcal{E}})^2\right)\tau + \Phi^{-1}(1-\alpha)(\sigma_{\pi_t}^{\mathcal{E}}\sqrt{\tau})\right] \leq \bar{E}_{t+\tau}
\end{aligned}
$$

We define the function g such that :

$$
g(\boldsymbol{\pi}_t) = \frac{E_t^\pi}{\bar{E}_{t+\tau}} \exp\left[\left(\mu_{\pi_t}^{\mathcal{E}} - \frac{1}{2}(\sigma_{\pi_t}^{\mathcal{E}})^2\right)\tau + \Phi^{-1}(1-\alpha)(\sigma_{\pi_t}^{\mathcal{E}}\sqrt{\tau})\right] - 1
$$

Then, the emission trajectory constraint reads:

$$
g(\boldsymbol{\pi}_t) \leq 0
$$

4.3.2 Constraint on the cumulative emission

We follow the same methodology for the constraint on cumulative emissions. The problem becomes:

$$
\mathbb{P}(CE_{0,T} > \bar{CE}_{0,T}|F_t) \leq \alpha, \forall t \in [0,T]
$$

At time t, we can write:

$$
CE_{0,T} = \sum_{s=0}^{T} E_s^\pi = \sum_{s=0}^{t} E_s^\pi + \sum_{s=t+1}^{T} E_s^\pi = CE_{0,t} + CE_{t+1,T}
$$

where $CE_{0,t}$ is known (ie: past cumulative financed emissions), so we are only interested in deriving $Y_{t+1,T}$: the cumulative future emissions. Again, we simplify by assuming π_t constant over the period, and we estimate total finance emissions if the position remain constant. The probability becomes:

$$
\begin{aligned}
\mathbb{P}(CE_{0,T} > \bar{CE}_{0,T}|F_t) &= \mathbb{P}(CE_{0,t} + CE_{t+1,T} > \bar{CE}_{0,T}|F_t) \\
&= \mathbb{P}(CE_{t+1,T} > \bar{CE}_{0,T} - CE_{0,t}|F_t)
\end{aligned}
$$

We can estimate the distribution of $CE_{t+1,T}$ by using the same the simplification as before. We define:

$$\begin{aligned}
CE_{t+1,T}(E_t^\pi, \pi_t) &= \sum_{\tau=1}^{T-t} E_{t+\tau}(E_t^\pi, \pi_t) \\
&= \sum_{\tau=1}^{T-t} E_t^\pi \exp\left[\left(\mu_{\pi_t}^{\mathcal{E}} - \frac{1}{2}(\sigma_{\pi_t}^{\mathcal{E}})^2\right)\tau + \sigma_{\pi_t}^{\mathcal{E}}[\tilde{W}_{t+\tau} - \tilde{W}_t]\right]
\end{aligned}$$

This is the sum between $t+1$ and T of emission when E_t^π and π_t are known and constant. The distribution of $CE_{t+1,T}(E_t^\pi, \pi_t)$ is a sum of log-normal distributed variables. We can approximate this distribution by using Monte Carlo simulation or use the Fenton-Wilkinson approximation Fenton (1960) [2]. We define m_1 and m_2 the first two moments of $CE_{t+1,T}(E_t^\pi, \pi_t)$. Their expressions are provided in Appendix A. We can approximate the distribution of $CE_{t+1,T}(E_t^\pi, \pi_t)$ by a log-normal distribution $LN(\mu_{FW}, \sigma_{FW}^2)$, with :

$$\begin{aligned}
\sigma_{FW,t}^2 &= \ln(1 + \frac{m_2}{m_1^2}) \\
\mu_{FW,t} &= \ln(m_1) - \frac{1}{2}\sigma_{FW,t}^2
\end{aligned}$$

Using these values, we can express the probability in the same fashion as before :

$$\begin{aligned}
\mathbb{P}(CE_{0,T} > \bar{CE}_{0,T}|F_t) &= \mathbb{P}(Y_{t+1,T} > \bar{CE}_{0,T} - CE_{0,t}|F_t) \\
&= 1 - \Phi\left(\frac{\log(\bar{CE}_{0,T} - CE_{0,t}) - \mu_{FW,t}}{\sigma_{FW,t}}\right)
\end{aligned}$$

This yields the following constraint:

$$\begin{aligned}
\mathbb{P}(CE_{0,T} > \bar{CE}_{0,T}|F_t) \leq& \alpha \Leftrightarrow \\
1 - \Phi\left(\frac{\log(\bar{CE}_{0,T} - CE_{0,t}) - \mu_{FW,t}}{\sigma_{FW,t}}\right) \leq& \alpha \Leftrightarrow \\
\exp\left(\mu_{FW,t} + \Phi^{-1}(1-\alpha)\sigma_{FW,t}\right) \leq& \bar{CE}_{0,T} - CE_{0,t}
\end{aligned}$$

We define the function h such that :

$$h(\boldsymbol{\pi}_t) = \frac{1}{\bar{CE}_{0,T} - CE_{0,t}} \exp\left(\mu_{FW,t} + \Phi^{-1}(1-\alpha)\sigma_{FW,t}\right) - 1$$

Then, the emission budget constraint reads:

$$h(\boldsymbol{\pi}_t) \leq 0$$

4.4 Emission-Constrained Portfolio Optimization

Let $\mathcal{A}$ be the set of admissible portfolios. We define two subsets of $\mathcal{A}$ for each $t \in [0, T]$:

$$\begin{aligned}
\mathcal{C}_t^1 &:= \{\pi_t \in \mathcal{A} : g(\pi_t) \leq 0,\ \pi_t \geq 0\}, \\
\mathcal{C}_t^2 &:= \{\pi_t \in \mathcal{A} : h(\pi_t) \leq 0,\ \pi_t \geq 0\},
\end{aligned}$$

where $\mathcal{C}_t^1$ represents the instantaneous emission constraints plus no-short-selling, and $\mathcal{C}_t^2$ represents cumulative emission constraints plus no-short-selling. Because the risk-free asset has no emission, both subsets are not empty.

[2]More complex methodologies could be implemented as this stage. However, we implement the Fenton-Wilkinson approximation to keep some degree of tractability.

Unconstrained Problem

The investor solves

$$\max_{\pi(\cdot)\in\mathcal{A}} \mathbb{E}_0\big[U(X_T^\pi)\big].$$

Following Korn and Nurkanovic (2023), we assume log-utility $U(x) = \ln x$. By Itô's lemma:

$$d\ln X_t^\pi = \Big(r + \pi_t^\top(\boldsymbol{\mu} - r\mathbf{1}) - \tfrac{1}{2}\pi_t^\top \boldsymbol{\Sigma}^W(\boldsymbol{\Sigma}^W)^\top \pi_t\Big)dt + \pi_t^\top \boldsymbol{\Sigma}^W dW_t.$$

Taking expectations removes the stochastic term:

$$\mathbb{E}[\ln X_T^\pi] = \ln X_0 + \int_0^T \Big(r + \pi_t^\top(\boldsymbol{\mu} - r\mathbf{1}) - \tfrac{1}{2}\pi_t^\top \boldsymbol{\Sigma}^W(\boldsymbol{\Sigma}^W)^\top \pi_t\Big)dt.$$

Maximizing the integrand pointwise yields the classical Merton solution:

$$\pi^* = (\boldsymbol{\Sigma}^W(\boldsymbol{\Sigma}^W)^\top)^{-1}(\boldsymbol{\mu} - r\mathbf{1}).$$

Constrained Problem

For log-utility, define the time-t pointwise objective:

$$f(\pi_t) := \pi_t^\top(\boldsymbol{\mu} - r\mathbf{1}) - \tfrac{1}{2}\pi_t^\top \boldsymbol{\Sigma}^W(\boldsymbol{\Sigma}^W)^\top \pi_t.$$

- **Instantaneous emission constraint:**

$$\pi_t^{\star,1} = \begin{cases} \pi^*, & \text{if } \pi^* \in \mathcal{C}_t^1, \\ \arg\max\limits_{\pi_t\in\mathcal{C}_t^1} f(\pi_t), & \text{otherwise.} \end{cases}$$

- **Cumulative emission constraint:**

$$\pi_t^{\star,2} = \begin{cases} \pi^*, & \text{if } \pi^* \in \mathcal{C}_t^2, \\ \arg\max\limits_{\pi_t\in\mathcal{C}_t^2} f(\pi_t), & \text{otherwise.} \end{cases}$$

These optimization problems are solved numerically using the Python implementation of the `IPOPT` solver Wächter and Biegler (2006).

5 Numerical analysis

5.1 Parameter Sensitivity of the constraint function for the case of a single risky asset

We begin by analysing the sensitivity of the constraint functions with respect to the model parameters in the case of a single risky asset ($d = 1$), with a small negative emission intensity drift. The baseline parameter values are reported in Table 1.

Parameter	**Value**
Number of risky assets, d	1
Risk-free rate, r	0.02
Expected return of risky asset, μ	0.05
Volatility of risky asset, σ	0.20
Emission intensity drift, b	-0.01
Emission intensity volatility, η	0.15
Correlation, ρ	0.5
Confidence level, α	0.05
Initial wealth, X_0	100
Initial emission intensity, I_0	50

Table 1: Model parameters for the case of a single risky asset.

For the emission trajectory constraint, we set the target level to $\bar{E}_{t+\tau} = 1210$ tCO_2-eq, while for the cumulative emissions we impose a budget $\bar{CE}_{0,T} = 231{,}274$ tCO_2-eq.[3] We fix the forecasting horizon to $\tau = 1$ month, so that the one-month emission forecast is directly compared with the target at the same horizon.

In the sensitivity analysis, each parameter is varied individually within $\pm 20\%$ of its baseline value, while keeping all other parameters fixed at their initial levels.

Emission trajectory constraint We evaluate the function g across different parameter values. By construction, $g > 0$ (resp. $g < 0$) indicates that financed emissions at time $t+1$ exceed (resp. fall below) the prescribed target. We find that an increase in the risky asset allocation, in initial portfolio wealth X_t, in initial intensity I_t, as well as in the drift and volatility of both the risky asset and the emission intensity process, all lead to higher forecasted emissions at horizon $t+1$. This behaviour is consistent with empirical evidence (see, e.g., FTSE Russell (LSEG) (2023)) and with the theoretical framework of Le Guenedal and Roncalli (2022), where portfolio decarbonization arises either through portfolio reallocation (changes in the control process π_t) or through asset self-decarbonization (negative drift in μ). Conversely, larger values of the confidence level α and of the target level $\bar{E}_{t+1}$ reduce the value of g.

[3] The trajectory constraint corresponds to a 10% reduction in portfolio-owned emissions under the unconstrained optimal strategy π^*. The cumulative budget is simulated as the expected sum of monthly emissions generated by a geometric Brownian motion with the same initial level, drift -0.05, and variance 0.01. In a real-world application, this value would be exogenously determined using scenario data.

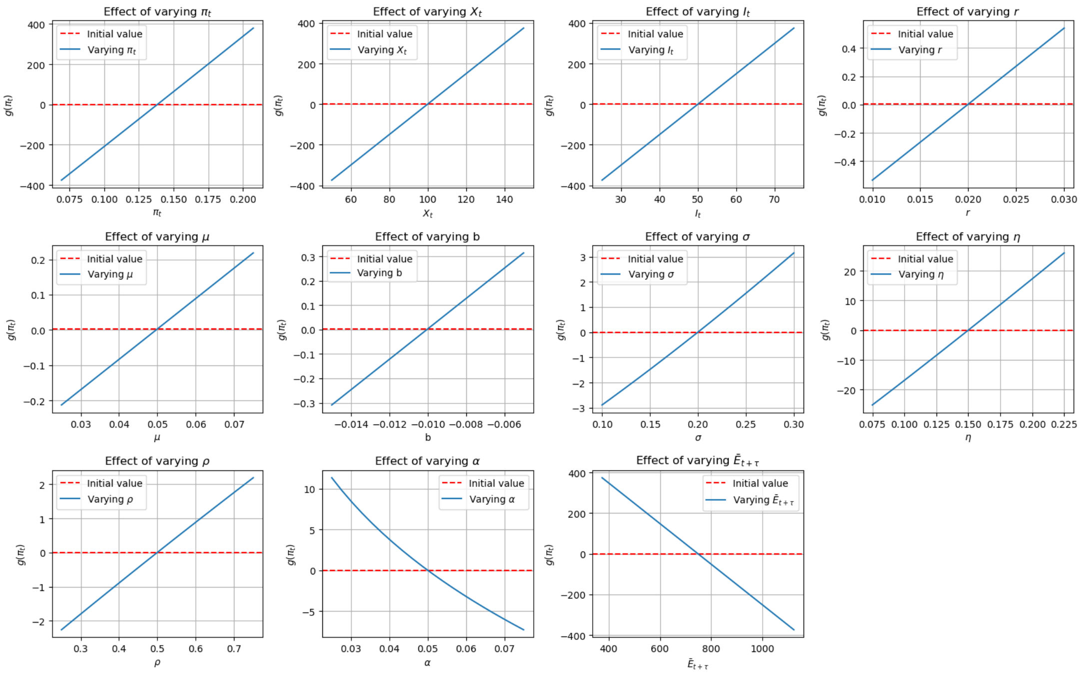

Figure 1: Sensitivity of the trajectory constraint function g to the model parameters.

Regarding the magnitude of the sensitivities, we observe that the share of risky asset, the initial portfolio wealth, the initial emission intensity, and the target level have the strongest influence on the value of g. This underscores the importance of accurately measuring both the latest observed portfolio-owned emissions and the target level when assessing compliance with the constraint.

Cumulative emissions constraint We conduct a similar analysis for the cumulative emissions constraint and evaluate the function h across different parameter values. The results are presented in Figure 2. The signs of the sensitivities are consistent with those obtained for the trajectory constraint. However, the magnitudes of the variations are substantially larger. This observation aligns with a well-known limitation of long-term, forward-looking metrics: their heightened sensitivity to model parameters. Such sensitivity may hinder the practical adoption of measures like the Implied Temperature Rise (ITR), as emphasized in Bouchet (2024).

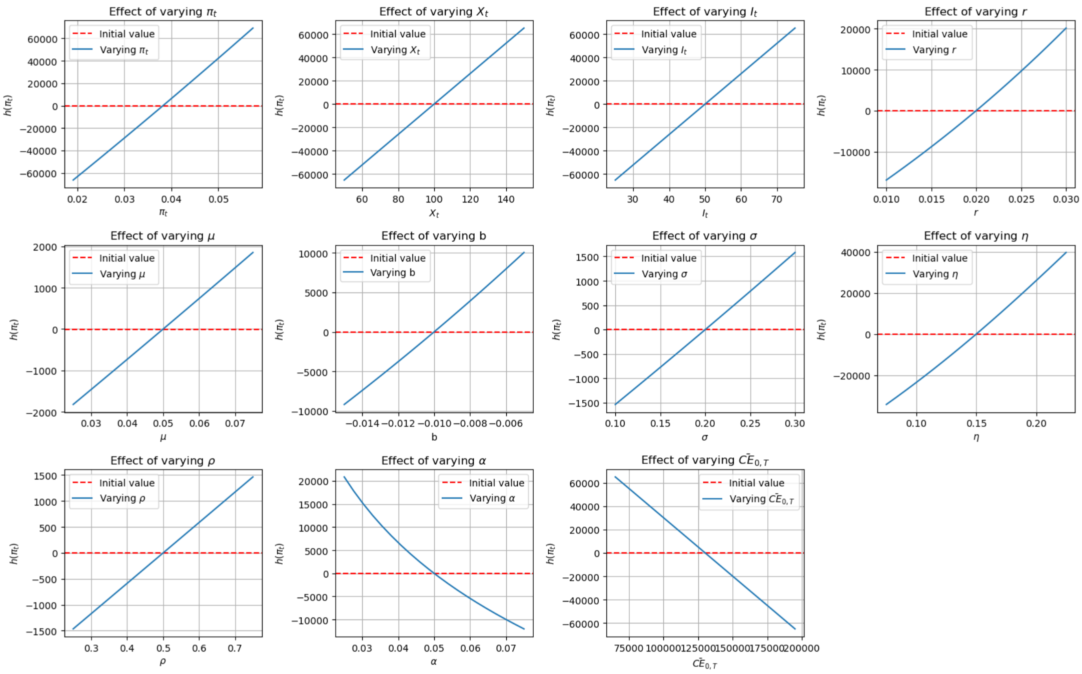

Figure 2: Sensitivity analysis of the function h to the different parameters

5.2 Emissions pathways under trajectory and budget constraints: single-asset portfolio

We analyse the evolution of the portfolio process together with the associated portfolio emissions, wealth, and cumulative emissions. The horizon is set to 25 years, with monthly rebalancing. The results are reported in Figure 3. First, we observe that both allocation strategies successfully comply with their respective constraints. Portfolio-owned emissions remain below the trajectory with 95% probability (lower left chart), while cumulative portfolio-owned emissions stay within the defined carbon budget over the investment horizon (lower right chart). The portfolio dynamics exhibit contrasting behaviours under the two types of constraints. When the constraint is imposed on the emission trajectory, the weight of the risky asset decreases gradually over time, adjusting to the emission intensity process and the trajectory limit. In contrast, under a cumulative emissions constraint, the portfolio initially reduces its risky exposure more sharply, reflecting the high uncertainty in projected cumulative emissions. As time progresses and realized emissions provide more information, uncertainty diminishes, allowing the portfolio to gradually increase its allocation to the risky asset.

It is interesting to notice that the portfolio process with both types of constraint dynamically reacts to the emission intensity process and asset price, allowing for a higher share of the risky asset when the emission intensity or the asset price is low [4].

[4] Note that the latter is specific to the setting with one risky asset: portfolio wealth is directly related to the price process of the risky asset. This conclusion might not hold in a more complex setting

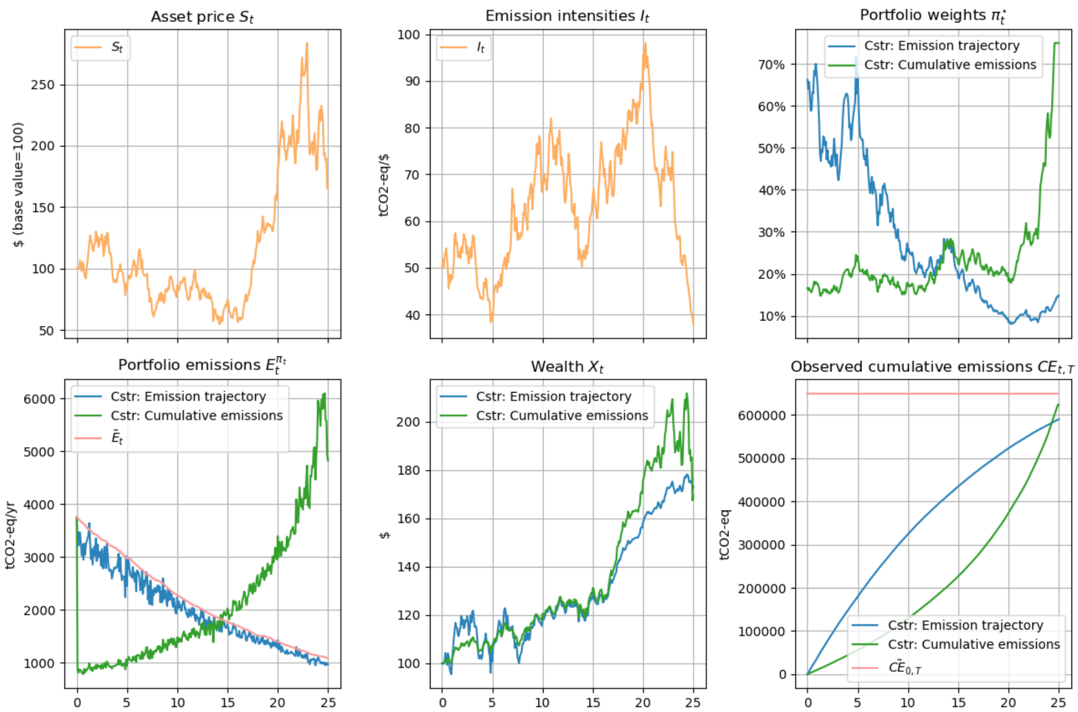

Figure 3: Portfolio process under a single risky asset

The cumulative emissions paths also differ in shape: they appear concave when constrained by the emission trajectory, but convex when constrained by cumulative emissions. This contrast highlights that cumulative constraints better align with the precautionary principle: as the horizon shortens and uncertainty decreases, more emissions can be allocated. Hence, this type of constraint is more robust to potential misspecification of the underlying assumptions.

In Appendix B, we present the same results but for the case of 100 simulations, allowing to validate the robustness of the results.

5.3 Weight comparison between emissions trajectory and budget for the case of a slow and fast decarbonization assets

We now illustrate our methodology in a two-asset setting. The first asset has a high initial emission intensity but exhibits rapid and predictable decarbonization, while the second asset shows a slow and uncertain decarbonization but starts with a lower initial intensity. This use case is designed to demonstrate how the two methodologies respond when the emissions intensity dynamics differs from their initial levels. The results are shown for 100 simulations.

Moreover, as observed in the previous case, the cumulative emissions constraint is highly restrictive at the beginning of the period. To mitigate this, we allow the confidence parameter α to start at 70% and decrease gradually to 5% by the end of the horizon. This approach allows to "discount" the high uncertainty in the projection of cumulative emissions at early stages, where a strict bound would be overly constraining. As emissions are realized over time, the bound progressively tightens, ensuring that the cumulative emission constraint is ultimately satisfied.

Parameter	Value	Description
d	2	Number of risky assets
r	0.02	Risk-free rate
μ	[0.05, 0.05]	Expected returns of the risky assets
Σ^W	$\begin{bmatrix} 0.20 & 0.00 \\ 0.00 & 0.20 \end{bmatrix}$	Volatility matrix of the risky assets (diagonal $\Rightarrow$ independent assets, each with 20% volatility)
b	$[-0.01, -0.04]$	Drift of emission intensities (asset 1 increases by 1% per unit time, asset 2 decreases by 4%)
Σ^B	$\begin{bmatrix} 0.10 & 0.00 \\ 0.00 & 0.05 \end{bmatrix}$	Volatility matrix of emission intensities (10% for asset 1, 5% for asset 2)
Σ^{WB}	$\begin{bmatrix} 0.5 & 0.2 \\ 0.2 & -0.5 \end{bmatrix}$	Correlation matrix between asset returns' Brownian motions and emission intensities' Brownian motions
α	0.05	Confidence level (e.g., 95% VaR or chance constraint)
I_0	[60, 80]	Initial intensities
S_0	[100, 100]	Initial asset prices

Table 2: Model parameters for the case of two risky assets.

Emission trajectory constraint Results are shown in Figure 4 and are consistent with the observations in the case of a single risky asset: the weights of both assets gradually decreases in order to follow the constraint on the emission trajectory. The portfolio weight of the fast-decarbonizing asset stabilizes around 30% while the weigh of the slow-decarbonizing asset until representing less than 10% of the portfolio.

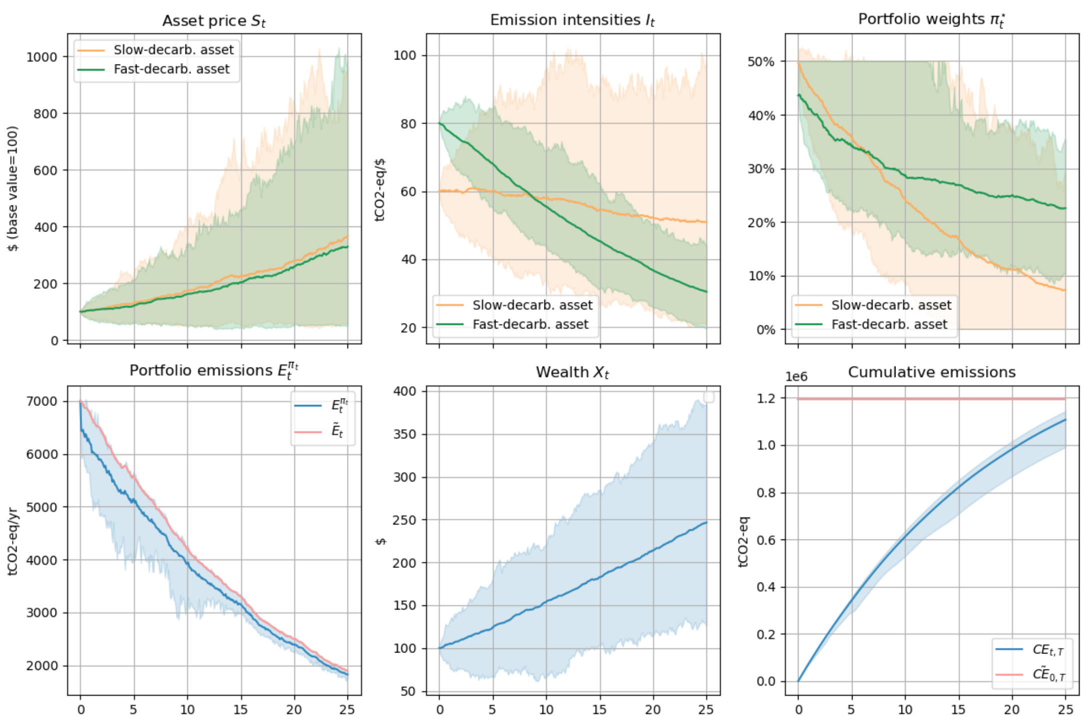

Figure 4: Portfolio dynamics for two assets under an emission trajectory constraint

The emission trajectory constraint seems short-sighted and does not incentivize engagement:

the slow-decarbonizing asset experiences significant divestment, while the fast-decarbonizing asset is favoured only when its emission intensity falls below that of the other asset.

Cumulative emissions constraint The cumulative emissions constraint is inherently more forward-looking, offering greater flexibility in the allocation of portfolio weights. As shown in Figure 5, while the weight of the slow-decarbonization asset is initially reduced, it increases again toward the end of the period, as the early reduction creates room for more emissions later on. Thus, although there is significant initial divestment, it does not persist throughout the horizon. The fast-decarbonization asset, on the other hand, after an initial reduction, shows a steady increase. Overall, after an early portfolio rebalancing, both assets are progressively given more weight. The portfolio-owned emissions chart further highlights that, even with an increased α at the start, the reduction efforts are still mostly front-loaded.

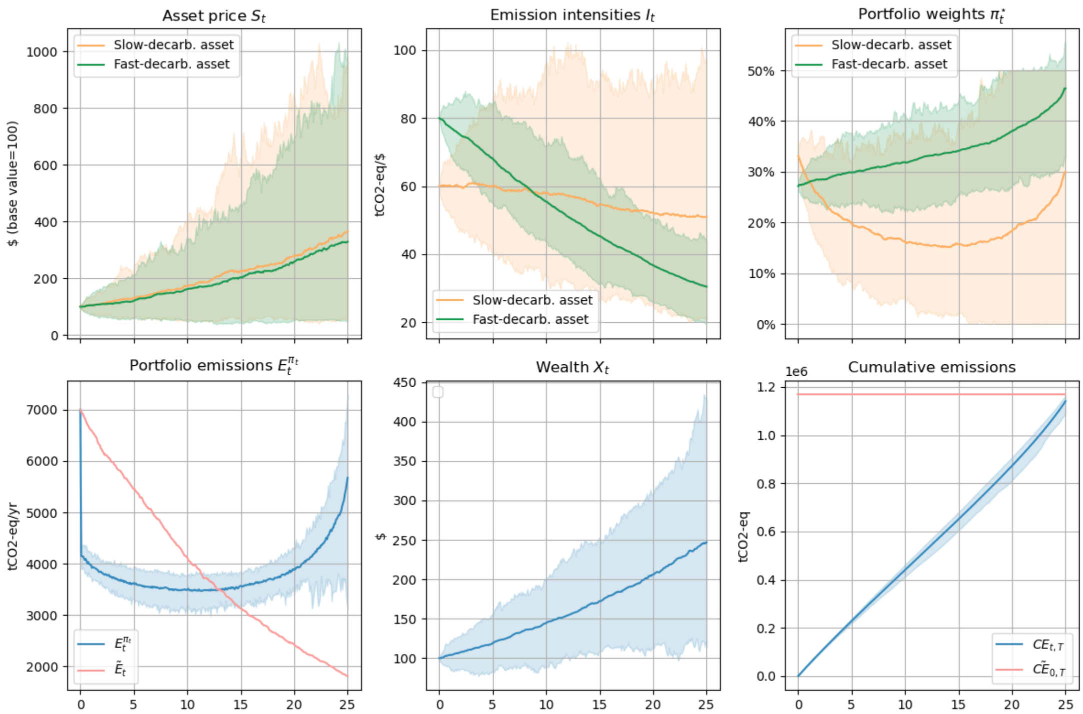

Figure 5: Portfolio dynamics for two assets under an cumulative emissions constraint

The end-of-period mean wealth attained with both strategies is relatively similar, with however a higher dispersion for the case of the cumulative emissions constraint.

6 Conclusion

As the effects of climate change become increasingly tangible, integrating climate-related constraints into portfolio investment strategies is essential. This integration not only supports the transition to a low-carbon economy but also mitigates exposure to high-emitting companies. In this paper, we propose a novel quantitative framework to evaluate the impact of such constraints within a dynamic portfolio optimization setting.

We model two distinct types of constraints:

- **Emission Trajectory Constraints:** These constraints predetermine the portfolio's annual owned emissions, aligning with the PAB approach. They emphasize current emissions and typically lead to a gradual divestment from carbon-intensive companies, without accounting for future emission trajectories.

- **Cumulative Emissions Constraints:** These constraints focus on total emissions over a specified horizon, resembling alignment-based strategies. They tend to front-load emission reductions, consistent with the precautionary principle. The resulting cumulative emission path is convex, allowing for potential further reductions in later periods.

While we do not expect this framework to be directly used as a portfolio management tool, we believe it can still provide valuable insights into the long-term implications of different types of carbon-related constraints. Indeed, it provides a structured approach to evaluate the impact of different decarbonization strategies, supports the integration of climate metrics into investment decision-making, and helps assess the trade-offs between short-term performance and long-term climate alignment. Furthermore, our framework enables the direct incorporation of carbon-related constraints into portfolio optimization and can be extended to other "portfolio-owned" physical quantities, such as fossil fuel reserves or biodiversity-related indicators (e.g., land use, water use). This adaptability makes it a valuable tool for assessing the broader impact of environmental metrics on investment strategies.

From a practical standpoint, future research could involve applying this framework to real-world data. However, scaling to a large number of assets (n potentially large) may pose computational challenges. Another promising direction is the inclusion of constraints on the "greenness" or transition relevance of companies, as explored in Slimane et al. (2023), which could enrich the framework by incorporating qualitative aspects of climate alignment. On the theoretical side, extending the utility function to more realistic forms would require dynamic optimization under several state variables: investor wealth and the emission intensity process of each asset. This would significantly increase the complexity of the problem. Additionally, replacing the emission intensity process with a direct process on emissions could better reflect available data on corporate emission targets. Finally, incorporating scenario uncertainty, as in Korn (2025), for example, through a Bayesian learning mechanism where the "true" scenario is progressively revealed could be particularly useful in this context.

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A Appendix: Derivation of the Fenton-Wilkinson parameters

The Fenton-Wilkinson method approximates the sum of log-normal distributed variables by a log-normal distribution, whose parameters are based on the empirical mean and variance of the sum. First, we simplify the notation as follows:

$$\mathbb{E}[CE_{t+1,T}(E_t^\pi, \pi_t)] = \sum_{\tau=1}^{T-t} \mathbb{E}[\tilde{E}_\tau]$$

For the empirical mean m_1, we have

$$\begin{aligned}
\mathbb{E}[\tilde{E}_\tau] &= E_t^\pi \exp\left(\left(\mu^{\mathcal{E}}_{\pi_t} - \frac{1}{2}(\sigma^{\mathcal{E}}_{\pi_t})^2\right)\tau\right)\mathbb{E}[\exp\left(\sigma^{\mathcal{E}}_{\boldsymbol{\pi}_t}(\tilde{W}_{t+\tau} - \tilde{W}_t)\right)] \\
&= E_t^\pi \exp\left(\mu^{\mathcal{E}}_{\boldsymbol{\pi}_t}\tau\right)
\end{aligned}$$

Leading to:

$$m_1 = \mathbb{E}[Y_{t+1,T}(E_t^\pi, \pi_t)] = \sum_{\tau=1}^{T-t} E_t^\pi \exp\left(\mu^{\mathcal{E}}_{\boldsymbol{\pi}_t}\tau\right)$$

For the variance, we use the fact that :

$$\mathrm{Var}(Y_{t+1,T}(E_t^\pi, \pi_t)) = \sum_{\tau=1}^{T-t}\left(\mathrm{Var}(\tilde{E}_\tau) + 2\sum_{\tau<s}\mathrm{Cov}(\tilde{E}_\tau, \tilde{E}_s)\right)$$

To compute the first part of the sum, we use:

$$\mathbb{E}[\tilde{E}_\tau^2] = (E_t^\pi)^2 \exp\left(2\mu^{\mathcal{E}}_{\boldsymbol{\pi}_t}\tau + (\sigma^{\mathcal{E}}_{\boldsymbol{\pi}_t})^2\tau\right)$$

Then :

$$\begin{aligned}
\mathrm{Var}(\tilde{E}_\tau) &= \mathbb{E}[\tilde{E}_\tau^2] - \mathbb{E}[\tilde{E}_\tau]^2 \\
&= (E_t^\pi)^2\left[\exp\left(2\mu^{\mathcal{E}}_{\boldsymbol{\pi}_t}\tau\right)\left(\exp\left((\sigma^{\mathcal{E}}_{\boldsymbol{\pi}_t})^2\tau\right) - 1\right)\right]
\end{aligned}$$

For the covariance, with $\tau < s$:

$$\begin{aligned}
\mathbb{E}(\tilde{E}_\tau\tilde{E}_s) &= (E_t^\pi)^2 \exp\left((\mu^{\mathcal{E}}_{\boldsymbol{\pi}_t} - \frac{1}{2}(\sigma^{\mathcal{E}}_{\pi_t})^2)(\tau+s)\right)\mathbb{E}\left[\exp\left(\sigma^{\mathcal{E}}_{\boldsymbol{\pi}_t}(\Delta\tilde{W}_\tau + \Delta\tilde{W}_s)\right)\right] \\
&= (E_t^\pi)^2 \exp\left((\mu^{\mathcal{E}}_{\boldsymbol{\pi}_t} - \frac{1}{2}(\sigma^{\mathcal{E}}_{\pi_t})^2)(\tau+s)\right)\exp\left(\frac{1}{2}(\sigma^{\mathcal{E}}_{\boldsymbol{\pi}_t})^2(\tau+s+2\min(\tau,s))\right) \\
&= (E_t^\pi)^2 \exp\left(\mu^{\mathcal{E}}_{\boldsymbol{\pi}_t}(\tau+s)\right)\exp\left((\sigma^{\mathcal{E}}_{\boldsymbol{\pi}_t})^2\tau\right)
\end{aligned}$$

And

$$\mathbb{E}[\tilde{E}_\tau]\mathbb{E}[\tilde{E}_s] = (E_t^\pi)^2\exp\left(\mu^{\mathcal{E}}_{\boldsymbol{\pi}_t}(\tau+s)\right)$$

Putting everything together:

$$\begin{aligned}
m_2 &= \mathrm{Var}(Y_{t+1,T}(E_t^\pi, \pi_t)) \\
&= \sum_{\tau=1}^{T-t}(E_t^\pi)^2\left[\exp\left(2\mu^{\mathcal{E}}_{\boldsymbol{\pi}_t}\tau\right)\left(\exp\left((\sigma^{\mathcal{E}}_{\boldsymbol{\pi}_t})^2\tau\right) - 1\right)\right] \\
&+ 2\sum_{\tau=1}^{T-t}\sum_{s<\tau}(E_t^\pi)^2\exp\left(\mu^{\mathcal{E}}_{\boldsymbol{\pi}_t}(\tau+s)\right)\left(\exp\left((\sigma^{\mathcal{E}}_{\boldsymbol{\pi}_t})^2\tau\right) - 1\right)
\end{aligned}$$

B Appendix: Application in the case of a single risky asset for 100 generation

Our methodology allows to rapidly scale in order to perform random generation and measure the robustness of the approach. Figure 6 presents the results for the case of 100 simulation. We observe the similar dynamics as in the simple case of a simulation, where the cumulative emission constraint initially lowers the investment in the risky asset. Over time, as cumulative emissions are revealed, more weight can be allocated to the risky asset. Conversely, for the case of the constraint on the emission trajectory, the share of the risky asset in the portfolio gradually decreases, in order to follow the decarbonization trajectory. Final wealth is relatively similar for both types of constraints.

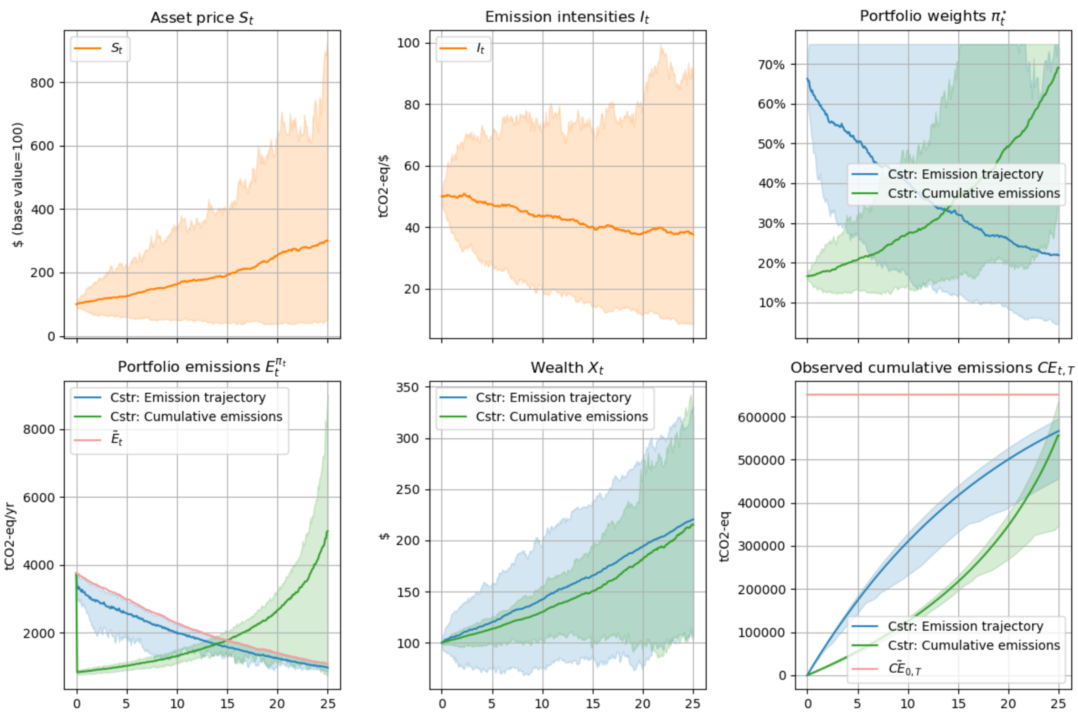

Figure 6: Results in the case of a single risky asset for 100 simulations